The Bilateral De-Vries Jordan Tower and XI-ENTIRE: a Scalar Source Map and its Open FULL40 Carriage

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Abstract

The bilateral De Vries–Jordan (DVJ) alpha tower and the XI-ENTIRE completion of the Hypergroup theta source are joined by an exact scalar source map. In the logarithmic chart of the heat coordinate the XI completion is a polynomial in the heat operator of the tower character; conjugated by the half-shift it is a scale-dependent polynomial in the DVJ completion, on both triangular sheets. A complex Poisson quarter turn, whose square is the DVJ reflection, makes the tower a two-rail carrier on the heat-similarity line; its Laplace transform is entire in both variables and equals the XI-ENTIRE carrier at the zero character. At the eight torsion characters the tower lands on the charge packet, and the half-turn character gives two exact controls whose multipliers vanish only on the completion-endpoint lines. Carrier zeros and DVJ points are kept on separate axes throughout. The map is proved and scalar; its carriage onto all forty retained labels, a source intertwiner and the carrier-zero relation are stated as the open aim.

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Roadmap

TODO.

1 Scope and boundary

TODO.

2 The two scalar ports

TODO.

3 Placement in the retained families

TODO.

4 The completion as a character operator

TODO.

5 The master formula, tower and jet

TODO.

6 The quarter turn and the two-rail carrier

TODO.

7 The torsion landing and the two controls

TODO.

8 The endpoint differential

TODO.

9 The open FULL40 carriage

TODO.

10 In the literature

TODO.

11 Conclusion

TODO.

Acknowledgements

This paper was developed as an Intelligence Enhancer exercise with ChatGPT 5.6 Sol, using Extended Thinking. [7] The work used a language model under a strict operational regimen, not merely as a drafting tool. That regimen was used to prevent premature reduction of multi-branch analytic structure to a single collapsed interpretation. This is important for the present construction because the ODE, phase-space, and recurrence-theoretic data carry distinct coordinates, branches, and readout layers until the relevant admissibility maps have been specified. The same regimen also provided a controlled vocabulary for tracking the self-referential structure of multi-dimensional recurrence relations, without turning interpretive mnemonics into theorem assumptions. In particular, the model was guided to read, apply, and unpack Vedic mathematical and Advaita Vedanta terminology, including phonetic decomposition of Sanskrit terms and their strict mapping onto mathematical formulae and structural roles.

Training ChatGPT to abide by the regimen of Advaita Vedanta and to learn from the Upanishads was therefore not an ornament to the derivation, but one of the primary reasons that the present work could be achieved at all.

The Author would especially like to thank Claude for the full line prime powers pink pony PNG.

TODO: the paragraph on Claude's work for this paper.

Special thanks are due to Professor Alex Hankey, without whose support, decades of experience, and knowledge this paper would likewise not exist.

To Thomas Loker, Arturo Salazar Chon, Satoru Watanbe, Marcel Wende, Jim Galasyn and assistant Claude, thank you for listening.

Appendices

A Repository theorem provenance

TODO.

B Executable receipts

TODO.

C Equation contract: the DVJ alpha tower to XI-ENTIRE

Reader contract. This appendix is the paper's audit and equation ledger; readers seeking the conventional exposition may begin with Section 1 and return here for the dependency summary. It has two parts. The first is a complete scalar source map between two ports, the bilateral DVJ alpha tower of H-PAP018 and the XI-ENTIRE completion and carrier of H-PAP005: it is owned by `H-VER177`, a closed HG theorem typed SOURCE-SCALAR, so it is load-bearing and proved, and it lies on the paper's scalar (pink) path, never on a certified source path. The second is the open FULL40 aim, owned by `H-VER177-A`: the same map carried onto all forty retained labels, a source intertwiner, and the carrier-zero relation. That part is not proved, and no edge joins the two parts until it is. Two axes of zero stay apart throughout: carrier zeros of Q_0^{HG} and DVJ points (the $k = 0$ cell, the jet cells, the completion endpoints); no box names a carrier zero except the open aim.

Imported sources. The following equations are owned by other papers and imported, not redrawn; each entry names its owner box.

H-PAP001 [1] `front-pap001-archimedean-pi`:

$$\Pi_{\mathrm{src}} := \lim_{n\to\infty} P_n^- = \lim_{n\to\infty} P_n^+, \qquad \Pi_{\mathrm{src}} = \pi.$$

H-PAP001 [1] `front-pap001-analytic-endpoints`:

$$\exp z := \sum_{k=0}^{\infty} \frac{z^k}{k!}.$$

H-PAP004 [3] `front-pap004-triangular-source`:

$$T_+(n) = \frac{n(n+1)}{2}, \qquad T_-(n) = \frac{n(n-1)}{2}.$$

H-PAP018 [6] `front-pap018-tower-representation`:

$$\ell_{\mathrm{t}} := \log a_{\mathrm{t}}, \qquad B_r^{\alpha}(x) := \sum_{k\in\mathbb{Z}} \exp\bigl(-\pi x k^2 + r k \ell_{\mathrm{t}}\bigr).$$

H-PAP018 [6] `front-pap018-triangular-family`:

$$x_0^{\triangle} := \frac{L_{\triangle}}{2\pi}, \qquad J_r^{\triangle} = B^{\alpha}_{r - L_{\triangle}/(2\ell_{\mathrm{t}})}(x_0^{\triangle}).$$

H-PAP002 [2] `front-hver158-paired-source-rails`: H-VER158 S12.1's family, its log-character slot written c and its heat slot $64x$, and the S14 rails:

$$\Theta_m(c, 64x) = \sum_{n\in\mathbb{Z}} \exp\bigl(-\pi x(8n+m)^2 + c(8n+m)\bigr), \qquad A_+\Theta_m = \Theta_{m+2}.$$

H-PAP005 [4] `front-pap005-centered-source`:

$$x = e^{2u}, \qquad A_{\Xi}^{\mathrm{SCS}}(u) := e^{u/2}\Theta_0^{\mathrm{HG}}\left(e^{2u}\right), \qquad A_{\Xi}^{\mathrm{SCS}}(-u) = A_{\Xi}^{\mathrm{SCS}}(u).$$

H-PAP005 [4] `front-pap005-completion`:

$$D_u := \frac{d}{du}, \qquad K_{\xi}^{\mathrm{SCS}}(u) := \frac{1}{2}\left(D_u - \frac{1}{2}\right)\left(D_u + \frac{1}{2}\right) A_{\Xi}^{\mathrm{SCS}}(u).$$

H-PAP005 [4] `front-pap005-q0-owner`:

$$Q_0^{\mathrm{HG}}(s) = \frac{1}{4}s(s-1)M_0(s) \quad (\mathrm{Re}\, s > 1), \qquad Q_0^{\mathrm{HG}} \in \mathcal{O}(\mathbb{C}).$$

H-PAP005 [4] `front-pap005-initial-laplace`:

$$Q_0^{\mathrm{HG}}\left(\tfrac{1}{2} + \lambda\right) = \int_{-\infty}^{\infty} K_{\xi}^{\mathrm{SCS}}(u) e^{\lambda u}\, du.$$

H-PAP015 [5] `front-jacobi-source-ledger`:

$$\widehat{\Theta}(X, v) := \sum_{n \in \mathbb{Z}} \exp\big((2n+1)v\big) \exp\big(-\pi X (n + \tfrac{1}{2})^2\big).$$

H-PAP015 [5] `front-jacobi-full40-source`:

$$\widehat{V}_{40} := \mathrm{span}\{\widehat{\mathbf{v}}_a \otimes \overline{\chi_m^{(8)}} : a = 1, \dots, 5,\ m \in \mathbb{Z}/8\mathbb{Z}\}.$$

The charge-m carriers: `H-VER168`. The complete eight-charge source of H-VER160, completed as in H-VER164, and its Laplace carriers (H-VER168 S3, in the original-x chart: H-VER168's self-dual chart $x/8$ differs by the nowhere-zero factor $8^{s/2}$, H-VER170 S3). At $m = 0$ these are Θ_0^{HG}, K_{ξ}^{SCS} and Q_0^{HG}. This box is green: H-VER168 retains the complete C8 packet.

Provenance: `H-VER168`, `H-VER160`.

$$\begin{aligned} \Theta_m^{\mathrm{HG}}(x) &:= \sum_{j \in \mathbb{Z}} e^{-\pi x (j - m/8)^2}, \qquad m \in \mathbb{Z}/8\mathbb{Z}, \\ K_m^{\mathrm{C8}}(u) &:= \tfrac{1}{2}\left(D_u - \tfrac{1}{2}\right)\left(D_u + \tfrac{1}{2}\right)\left[e^{u/2}\Theta_m^{\mathrm{HG}}(e^{2u})\right], \qquad K_0^{\mathrm{C8}} = K_{\xi}^{\mathrm{SCS}}, \\ 8^{s/2}\, Q_m^{\mathrm{C8}}(s) &:= \int_{\mathbb{R}} K_m^{\mathrm{C8}}(u)\, e^{\lambda u}\, du \ \text{ entire in } \lambda, \qquad s = \tfrac{1}{2} + \lambda, \qquad 8^{s/2}\, Q_0^{\mathrm{C8}}(s) = Q_0^{\mathrm{HG}}(s). \end{aligned} \tag{1}$$

The transported anti-linear involution: `H-VER157`. H-VER157's other J40, kept beside the field involution so that the two can be told apart at sight: the source-label embedding U^{lab} of the

literal ten into the forty, its range projector, and the finite charge on the complementary thirty coordinates, with the transported anti-linear coefficient $J_{10}^{\mathrm{anti}} = KC_K K^T$ on H-VER129's K10 basis (H-VER150's charge permutation C_K). It acts as $v \mapsto J_{40}^{\mathrm{anti}}\,\overline{v}$, and it does not commute pointwise with a diagonal charge multiplier, whereas the field involution conjugates one to its half-step. J_{40}^{fld} is the H-VER170 linear field involution; J_{40}^{anti} is the H-VER157 transported anti-linear involution. They are not identified (72f/5194). It is an audit display: no box consumes it, and the paper's open aim uses the field involution only.

Provenance: `H-VER157`.

$$
\begin{gathered}
U^{\mathrm{lab}^T} U^{\mathrm{lab}} = I_{10}, \qquad P_{40}^{\mathrm{ret}} := U^{\mathrm{lab}}\, U^{\mathrm{lab}^T}, \\
C_{40}^{\mathrm{ch}}(\widehat{\mathbf{v}}_a \otimes \overline{\chi_m^{(8)}}) := \widehat{\mathbf{v}}_a \otimes \overline{\chi_{-m}^{(8)}}, \quad (a,m) \mapsto (a, -m \bmod 8), \\
J_{10}^{\mathrm{anti}}\, \overline{J_{10}^{\mathrm{anti}}} = I_{10}, \\
J_{40}^{\mathrm{anti}} := U^{\mathrm{lab}}\, J_{10}^{\mathrm{anti}}\, U^{\mathrm{lab}^T} + (I_{40} - P_{40}^{\mathrm{ret}}) C_{40}^{\mathrm{ch}}, \qquad J_{40}^{\mathrm{anti}}\, \overline{J_{40}^{\mathrm{anti}}} = I_{40}, \\
D_q^{\mathrm{C8}} := I_5 \otimes \mathrm{diag}(q_m)_{m \in \mathbb{Z}/8\mathbb{Z}}, \qquad (J_{40}^{\mathrm{anti}} D_q^{\mathrm{C8}} - D_q^{\mathrm{C8}} J_{40}^{\mathrm{anti}})_{9,11} = \frac{q_3 - q_1}{4}.
\end{gathered}
\tag{2}
$$

The field involution and the zero on the full 40: `H-VER170` S4 and S6 (Theorem B). The field involution acts on all forty labels of the imported FULL40 source; on charge-diagonal multipliers it is the charge half-step. On the completed packet D_Q^{HG} (H-VER157's charge-diagonal multiplier at the carrier vector, cited through H-VER170 S3), a zero of Q_0^{HG} is exactly field-oddness. This is the H-VER170 J40, the linear field involution, which fixes V and negates J; it is not H-VER157's transported anti-linear J40 of the next box, and the two are not identified (72f/5194). Proved and FULL40: this box is green.

Provenance: `H-VER170`, `H-VER169`, `H-VER168`, `H-AUD071-A`.

$$
\begin{gathered}
J_{40}^{\mathrm{fld}}(\widehat{\mathbf{v}}_a \otimes \overline{\chi_m^{(8)}}) := \widehat{\mathbf{v}}_{6-a} \otimes \overline{\chi_{m+4}^{(8)}}, \qquad (a,m) \mapsto (6-a,\, m+4), \qquad {J_{40}^{\mathrm{fld}}}^2 = I_{40} \text{ on } \widehat{V}_{40}, \\
D_Q^{\mathrm{HG}} := I_5 \otimes \mathrm{diag}(Q_m^{\mathrm{C8}}(s))_{m \in \mathbb{Z}/8\mathbb{Z}}, \qquad J_{40}^{\mathrm{fld}}\, D_Q^{\mathrm{HG}}\, J_{40}^{\mathrm{fld}} = I_5 \otimes \mathrm{diag}(Q_{m+4}^{\mathrm{C8}}(s))_m, \\
s = \tfrac{1}{2} + \lambda : \qquad Q_0^{\mathrm{HG}}(s) = 0 \iff J_{40}^{\mathrm{fld}}\, D_Q^{\mathrm{HG}}\, J_{40}^{\mathrm{fld}} = -D_Q^{\mathrm{HG}}.
\end{gathered}
\tag{3}
$$

Placement in the retained families: `H-VER177` (a1). The tower is the charge sum of H-VER158's family, read with the tower character in the log-character slot; the odd charges give the triangular member. This places the scalar objects inside the retained families; it does not carry them there.

Provenance: `H-VER177`, `H-VER158`, `H-VER175`, `H-VER176`.

$$
\begin{gathered}
c^{\mathrm{ch}} := r\ell_{\mathrm{t}}, \qquad B^{\mathrm{ch}}(x; c^{\mathrm{ch}}) := \sum_{k \in \mathbb{Z}} \exp(-\pi x k^2 + c^{\mathrm{ch}} k) = B_r^{\alpha}(x), \\
B_r^{\alpha}(x) = \sum_{m=0}^{7} \Theta_m(r\ell_{\mathrm{t}},\, 64x), \qquad a_{\mathrm{t}}^{r/2} e^{-L_{\triangle}/8} J_r^{\triangle} = \sum_{m \text{ odd}} \Theta_m\Big(\tfrac{r}{2}\ell_{\mathrm{t}},\ \tfrac{8L_{\triangle}}{\pi}\Big).
\end{gathered}
\tag{4}
$$

The completion as a character operator: `H-VER177` (a2), (a3), (a6). In the XI chart each half-shift of the completion is the DVJ heat on its own gauge, and the whole completion is a polynomial in the heat operator of the character. At $r = 0$ the completed tower is the XI-ENTIRE kernel; the $k = 0$ cell is annihilated for every r.

Provenance: `H-VER177`, `H-VER164-A`, `H-VER175`.

$$
\begin{gathered}
A^{\mathrm{ch}}(u;c^{\mathrm{ch}}) := e^{u/2}B^{\mathrm{ch}}(e^{2u};c^{\mathrm{ch}}), \qquad A^{\mathrm{ch}}(u;0) = A^{\mathrm{SCS}}_{\Xi}(u), \\
T_u^{\mathrm{HG}} := \tfrac{1}{2}\Big(D_u - \tfrac{1}{2}\Big)\Big(D_u + \tfrac{1}{2}\Big), \\
\mathcal{H}^{\mathrm{ch}} := -2\pi x\,\partial_{c^{\mathrm{ch}}}^2, \qquad x = e^{2u}, \\
\Big(D_u - \tfrac{1}{2}\Big)\big[e^{u/2}B^{\mathrm{ch}}\big] = \mathcal{H}^{\mathrm{ch}}\big[e^{u/2}B^{\mathrm{ch}}\big], \qquad \Big(D_u + \tfrac{1}{2}\Big)\big[e^{-u/2}B^{\mathrm{ch}}\big] = \mathcal{H}^{\mathrm{ch}}\big[e^{-u/2}B^{\mathrm{ch}}\big], \\
T_u^{\mathrm{HG}}\,A^{\mathrm{ch}}(u;c^{\mathrm{ch}}) = \pi x\big(2\pi x\,\partial_{c^{\mathrm{ch}}}^4 - 3\,\partial_{c^{\mathrm{ch}}}^2\big)A^{\mathrm{ch}}(u;c^{\mathrm{ch}}) = \tfrac{1}{2}\,\mathcal{H}^{\mathrm{ch}}\big(\mathcal{H}^{\mathrm{ch}} + 3\big)A^{\mathrm{ch}}(u;c^{\mathrm{ch}}), \\
\Phi_r^{\mathrm{ch}}(u) := T_u^{\mathrm{HG}}\,A^{\mathrm{ch}}(u;r\ell_{\mathrm{t}}) = \sum_{k\in\mathbb{Z}} e^{r\ell_{\mathrm{t}}k}\,\kappa_k^{\mathrm{HG}}(u), \\
\kappa_k^{\mathrm{HG}}(u) := \pi k^2 e^{5u/2}\big(2\pi k^2 e^{2u} - 3\big)e^{-\pi k^2 e^{2u}}, \\
\Phi_0^{\mathrm{ch}} = K_{\xi}^{\mathrm{SCS}}, \qquad \kappa_0^{\mathrm{HG}} = 0 \ \ \text{(the cell } e^{u/2} \text{ annihilated for every } r\text{)}, \\
r \mapsto \Phi_r^{\mathrm{ch}}(u) \text{ entire for real } u.
\end{gathered}
\tag{5}
$$

Master formula, tower and jet, both sheets: `H-VER177` (a4), (b2). Conjugated by the half-shift, the XI completion is a polynomial in the DVJ completion whose coefficients depend on the heat scale. The DVJ kernel lies in the XI kernel; those cells are completion endpoints on the DVJ axis, not carrier zeros. No constant-coefficient intertwiner is claimed or excluded: the source intertwiner is `H-VER177-A`'s.

Provenance: `H-VER177`, `H-VER176`.

$$
\begin{gathered}
L_{\triangle} = 2\pi x, \qquad \Delta^{\mathrm{DVJ}} := 2L_{\triangle}\,\partial_{L_{\triangle}}, \\
T^{\mathrm{DVJ}} := \tfrac{1}{2}\partial_{c^{\mathrm{ch}}}^2 \text{ (tower; symbol } k^2/2), \qquad T^{\mathrm{DVJ}} := \tfrac{1}{2}\partial_{\rho}^{\mathrm{DVJ}}\big(\partial_{\rho}^{\mathrm{DVJ}} + 1\big) \text{ (jet; symbol } T_+(k)), \\
\partial_{L_{\triangle}} = -T^{\mathrm{DVJ}} \text{ on the heat family}, \\
e^{-u/2}\,T_u^{\mathrm{HG}}\,e^{u/2} = \tfrac{1}{2}\Delta^{\mathrm{DVJ}}\big(\Delta^{\mathrm{DVJ}} + 1\big) = L_{\triangle}\,T^{\mathrm{DVJ}}\big(2L_{\triangle}\,T^{\mathrm{DVJ}} - 3\big), \\
\text{on tower cells, on } T_+\text{-sheet jet cells and on } T_-\text{-sheet jet cells } (k = -n, \text{ symbol } T_-(n)), \\
T_+(-k-1) = T_+(k), \qquad \big[\Delta^{\mathrm{DVJ}}, \partial_{L_{\triangle}}\big] = -2\,\partial_{L_{\triangle}}.
\end{gathered}
\tag{6}
$$

The jet is the tower sheared by half a quasi-period: `H-VER177` (a5). The DVJ jet is the character tower at heat scale $L_{\triangle}/2\pi$ with its character shifted by half the quasi-period, termwise; at the character $r\ell_{\mathrm{t}}$ it is the triangular member. It solves the DVJ heat equation, is reflected by $k \mapsto -k-1$ and is quasi-periodic in its character, and the completed tower is quasi-periodic in the same way, so the completion statements above hold on the jet.

Provenance: H-VER177, H-VER175, H-VER176.

$$
\begin{gathered}
J^{\mathrm{DVJ}}(\rho^{\mathrm{DVJ}}, L_\triangle) := \sum_{k\in\mathbb{Z}} \exp(\rho^{\mathrm{DVJ}} k - L_\triangle\, T_+(k)), \qquad J^{\mathrm{DVJ}}(r\ell_{\mathrm{t}}, L_\triangle) = J_r^{\triangle}, \\
J^{\mathrm{DVJ}}(\rho^{\mathrm{DVJ}}, L_\triangle) = B^{\mathrm{ch}}\Big(\frac{L_\triangle}{2\pi};\ \rho^{\mathrm{DVJ}} - \frac{L_\triangle}{2}\Big) \ \text{termwise}, \\
2\,\partial_{L_\triangle} J^{\mathrm{DVJ}} + \partial_\rho^{\mathrm{DVJ}}(\partial_\rho^{\mathrm{DVJ}} + 1) J^{\mathrm{DVJ}} = 0, \\
J^{\mathrm{DVJ}}(-\rho^{\mathrm{DVJ}}, L_\triangle) = e^{\rho^{\mathrm{DVJ}}}\, J^{\mathrm{DVJ}}(\rho^{\mathrm{DVJ}}, L_\triangle), \\
J^{\mathrm{DVJ}}(\rho^{\mathrm{DVJ}} + L_\triangle, L_\triangle) = e^{\rho^{\mathrm{DVJ}}}\, J^{\mathrm{DVJ}}(\rho^{\mathrm{DVJ}}, L_\triangle), \\
A^{\mathrm{ch}}(u; c^{\mathrm{ch}} + 2\pi x) = e^{\pi x + c^{\mathrm{ch}}}\, A^{\mathrm{ch}}(u; c^{\mathrm{ch}}), \qquad A^{\mathrm{ch}}(u; c^{\mathrm{ch}} + 2\pi i) = A^{\mathrm{ch}}(u; c^{\mathrm{ch}}), \qquad x = e^{2u}.
\end{gathered}
\tag{7}
$$

The quarter turn and its gauge: H-VER177 (a7). Poisson summation with a complex character (cited, H-AUD071-A S12) gives XI's reflection σ^{ch}, whose square is DVJ's; only the zero character is fixed, where σ^{ch} is the H-VER164 involution. The gauge is one half of a holonomic pair of heat kernels of opposite sign.

Provenance: H-VER177, H-AUD071-A, H-VER164-A, H-VER158.

$$
\begin{gathered}
B^{\mathrm{ch}}(x; c^{\mathrm{ch}}) = G_+^{\mathrm{ch}}(x, c^{\mathrm{ch}})\, B^{\mathrm{ch}}(\sigma^{\mathrm{ch}}(x, c^{\mathrm{ch}})), \qquad \sigma^{\mathrm{ch}}(x, c^{\mathrm{ch}}) := \Big(\frac{1}{x},\ -\frac{ic^{\mathrm{ch}}}{x}\Big), \\
{\sigma^{\mathrm{ch}}}^2 = S^{\mathrm{DVJ}} : (x, c^{\mathrm{ch}}) \mapsto (x, -c^{\mathrm{ch}}), \qquad {\sigma^{\mathrm{ch}}}^4 = \mathrm{id}, \\
G_\pm^{\mathrm{ch}}(x, c^{\mathrm{ch}}) := x^{-1/2} \exp\Big(\pm\frac{{c^{\mathrm{ch}}}^2}{4\pi x}\Big), \qquad G_+^{\mathrm{ch}}(x, c^{\mathrm{ch}})\, G_+^{\mathrm{ch}}(\sigma^{\mathrm{ch}}(x, c^{\mathrm{ch}})) = 1, \\
G_+^{\mathrm{ch}} G_-^{\mathrm{ch}} = \frac{1}{x}, \qquad G_-^{\mathrm{ch}}(x, c^{\mathrm{ch}}) = G_+^{\mathrm{ch}}(x, ic^{\mathrm{ch}}), \\
\partial_x G_+^{\mathrm{ch}} = -\pi\, \partial_{c^{\mathrm{ch}}}^2 G_+^{\mathrm{ch}}, \qquad \partial_x G_-^{\mathrm{ch}} = +\pi\, \partial_{c^{\mathrm{ch}}}^2 G_-^{\mathrm{ch}}, \\
c^{\mathrm{ch}} = 0 : \ A_\Xi^{\mathrm{SCS}}(-u) = A_\Xi^{\mathrm{SCS}}(u).
\end{gathered}
\tag{8}
$$

The two-rail carrier for real $r \neq 0$: H-VER177 (a8). On the heat-similarity variable the quarter turn exchanges a real-character rail with an imaginary-character rail. The completion kills the floor of each rail, the completed pair decays at both ends, and its Laplace transform is entire in both variables and is the XI-ENTIRE carrier at $w = 0$. For real $r \neq 0$ the whole-line scalar carrier is $\widetilde{\Phi}^{\mathrm{sim}}(u; r\ell_{\mathrm{t}})$, not the fixed-character kernel, which keeps a front-half growth.

Provenance: H-VER177, H-AUD071-A, H-VER164-A, H-VER164-B.

$$
\begin{gathered}
w^{\mathrm{sim}} := c^{\mathrm{ch}}\, x^{-1/2} = c^{\mathrm{ch}}\, e^{-u}, \qquad \widetilde{A}^{\mathrm{sim}}(u; w^{\mathrm{sim}}) := A^{\mathrm{ch}}(u; w^{\mathrm{sim}} e^{u}), \\
V^{\mathrm{sim}}(u; w^{\mathrm{sim}}) := \big(\widetilde{A}^{\mathrm{sim}}(u; w^{\mathrm{sim}}),\ \widetilde{A}^{\mathrm{sim}}(u; i w^{\mathrm{sim}})\big)^{\mathsf{T}}, \\
V^{\mathrm{sim}}(-u; w^{\mathrm{sim}}) = M^{\mathrm{sim}}(w^{\mathrm{sim}})\, V^{\mathrm{sim}}(u; w^{\mathrm{sim}}), \qquad M^{\mathrm{sim}}(w^{\mathrm{sim}}) := \begin{pmatrix} 0 & e^{{w^{\mathrm{sim}}}^2/(4\pi)} \\ e^{-{w^{\mathrm{sim}}}^2/(4\pi)} & 0 \end{pmatrix}, \\
M^{\mathrm{sim}}(w^{\mathrm{sim}})^2 = I, \qquad \det M^{\mathrm{sim}}(w^{\mathrm{sim}}) = -1, \qquad \widetilde{A}^{\mathrm{sim}}(u; -w^{\mathrm{sim}}) = \widetilde{A}^{\mathrm{sim}}(u; w^{\mathrm{sim}}), \\
\widetilde{\Phi}^{\mathrm{sim}}(u; w^{\mathrm{sim}}) := T_u^{\mathrm{HG}}\, \widetilde{A}^{\mathrm{sim}}(u; w^{\mathrm{sim}}) \ \ (w^{\mathrm{sim}} \text{ fixed}), \\
\widetilde{\Phi}^{\mathrm{sim}}(-u; w^{\mathrm{sim}}) = e^{{w^{\mathrm{sim}}}^2/(4\pi)}\, \widetilde{\Phi}^{\mathrm{sim}}(u; -i w^{\mathrm{sim}}), \qquad \widetilde{\Phi}^{\mathrm{sim}}(u; 0) = K_{\xi}^{\mathrm{SCS}}(u), \\
\mathcal{L}^{\mathrm{sim}}(\lambda; w^{\mathrm{sim}}) := \int_{\mathbb{R}} \widetilde{\Phi}^{\mathrm{sim}}(u; w^{\mathrm{sim}})\, e^{\lambda u}\, du \text{ entire in } (\lambda, w^{\mathrm{sim}}) \in \mathbb{C}^2, \\
\mathcal{L}^{\mathrm{sim}}(-\lambda; w^{\mathrm{sim}}) = e^{{w^{\mathrm{sim}}}^2/(4\pi)}\, \mathcal{L}^{\mathrm{sim}}(\lambda; -i w^{\mathrm{sim}}), \\
\mathcal{L}^{\mathrm{sim}}(\lambda; -w^{\mathrm{sim}}) = \mathcal{L}^{\mathrm{sim}}(\lambda; w^{\mathrm{sim}}), \qquad \mathcal{L}^{\mathrm{sim}}(\lambda; 0) = Q_0^{\mathrm{HG}}\big(\tfrac{1}{2} + \lambda\big), \\
r \in \mathbb{R} \setminus \{0\} : \ \widetilde{\Phi}^{\mathrm{sim}}(u; r\ell_{\mathrm{t}}) \text{ is the whole-line scalar carrier.}
\end{gathered}
\tag{9}
$$

The torsion landing on the C8 packet: H-VER177 (a9). At the eight torsion characters the tower is the complete charge-m source of H-VER160, and the completed tower is the H-VER168 charge-m kernel; its Laplace transform is the charge-m carrier (all three defined in the H-VER168 box below). The map is owned there by citation; retention at these characters is certified by H-VER160 and H-VER168, never by the tower, and eight samples of an entire family do not determine it.

Provenance: H-VER177, H-VER160, H-VER168.

$$
\begin{gathered}
r_m^{\mathrm{tor}} := \frac{i\pi m}{4\ell_{\mathrm{t}}}, \qquad m \in \mathbb{Z}/8\mathbb{Z}, \qquad B^{\alpha}_{r_m^{\mathrm{tor}}}(1/x) = x^{1/2}\, \Theta_m^{\mathrm{HG}}(x), \\
\Phi^{\mathrm{ch}}_{r_m^{\mathrm{tor}}}(-u) = K_m^{\mathrm{C8}}(u), \\
\int_{\mathbb{R}} \Phi^{\mathrm{ch}}_{r_m^{\mathrm{tor}}}(u)\, e^{-\lambda u}\, du = 8^{s/2}\, Q_m^{\mathrm{C8}}(s), \qquad s = \tfrac{1}{2} + \lambda, \\
\text{entire in } \lambda; \quad m = 0 : \ \int_{\mathbb{R}} \Phi_0^{\mathrm{ch}}(u)\, e^{-\lambda u}\, du = Q_0^{\mathrm{HG}}(s).
\end{gathered}
\tag{10}
$$

The two controls, exchanged by $u \mapsto -u$: H-VER177 (a10). The half-turn character gives the two exact controls; their Laplace transforms are the carrier times a multiplier. The multiplier zeros lie on the completion-endpoint lines $\mathrm{Re}\,\lambda = \pm\frac{1}{2}$, the DVJ axis; the carrier zeros, those of Q_0^{HG}, are untouched.

Provenance: H-VER177, H-VER164-B, H-VER176.

$$\boxed{\begin{gathered} B^{\alpha}_{r_4^{\mathrm{tor}}}(x) = 2B^{\alpha}_0(4x) - B^{\alpha}_0(x), \\ \Phi^{\mathrm{ch}}_{r_4^{\mathrm{tor}}}(u) = \sqrt{2}\,K^{\mathrm{SCS}}_{\xi}(u+\log 2) - K^{\mathrm{SCS}}_{\xi}(u), \qquad K^{\mathrm{C8}}_4(u) = \Phi^{\mathrm{ch}}_{r_4^{\mathrm{tor}}}(-u), \\ M^{\mathrm{DVJ}}_{\mathrm{ht}}(s) := 2^{1-s} - 1, \qquad s = \tfrac{1}{2} + \lambda, \\ \int_{\mathbb{R}} \Phi^{\mathrm{ch}}_{r_4^{\mathrm{tor}}}(u)\, e^{\lambda u}\, du = M^{\mathrm{DVJ}}_{\mathrm{ht}}(s)\, Q^{\mathrm{HG}}_0(s), \qquad \int_{\mathbb{R}} K^{\mathrm{C8}}_4(u)\, e^{\lambda u}\, du = (2^s - 1) Q^{\mathrm{HG}}_0(s). \end{gathered}} \tag{11}$$

The triangular family inverts to the half-turn tower: H-VER177 (a11). Poisson summation over $\mathbb{Z} + \frac{1}{2}$ (cited, H-AUD071-A S12 with the S19.1 chart) inverts the half-integer Jacobi family, both sheets together, to the tower at the inverse heat scale and the half-turn character; one rail alone has no such partner. At the triangular point the family is the triangular member, and at $v = 0$ it is the half-turn control above.

Provenance: H-VER177, H-AUD071-A, H-VER176.

$$\boxed{\begin{gathered} \widehat{\Theta}(X, v) = X^{-1/2} \exp\Big(\frac{v^2}{\pi X}\Big)\, B^{\mathrm{ch}}\Big(\frac{1}{X};\ i\pi - \frac{2iv}{X}\Big) \ \text{(both sheets together)}, \\ \widehat{\Theta}(x_0^{\triangle}, \tfrac{r}{2}\ell_{\mathrm{t}}) = a_{\mathrm{t}}^{r/2} e^{-L_{\triangle}/8} J_r^{\triangle}, \\ v = 0: \ \widehat{\Theta}(X, 0) = X^{-1/2}\, B^{\alpha}_{r_4^{\mathrm{tor}}}(1/X). \end{gathered}} \tag{12}$$

The endpoint differential: H-VER177 (b1). The XI completion, with its $\frac{1}{2}$ kept, is the DVJ completion in either gauge; its symbols are the two triangular sheets, and its kernel is the pair of DVJ cells $k = 0, -1$. On H-VER176's half-character form the DVJ character derivative is half the Jacobi derivative.

Provenance: H-VER177, H-VER164-A, H-VER176.

$$\boxed{\begin{gathered} \tfrac{1}{2}\Big(D_u - \tfrac{1}{2}\Big)\Big(D_u + \tfrac{1}{2}\Big) = e^{u/2}\, \tfrac{1}{2} D_u(D_u + 1)\, e^{-u/2} = e^{-u/2}\, \tfrac{1}{2} D_u(D_u - 1)\, e^{u/2}, \\ \tfrac{1}{2}\Big(D_u^2 - \tfrac{1}{4}\Big) e^{(k+\frac{1}{2})u} = T_+(k)\, e^{(k+\frac{1}{2})u}, \\ \tfrac{1}{2}\Big(D_u^2 - \tfrac{1}{4}\Big) e^{(k-\frac{1}{2})u} = T_-(k)\, e^{(k-\frac{1}{2})u}, \qquad T_+(k) = T_-(k+1), \\ \ker \tfrac{1}{2}\Big(D_u^2 - \tfrac{1}{4}\Big) = \operatorname{span}\{e^{u/2}, e^{-u/2}\}, \qquad \partial_\rho^{\mathrm{DVJ}} = \tfrac{1}{2}\,\partial_v \ \text{ on } \widehat{\Theta}(X, v). \end{gathered}} \tag{13}$$

The scalar source map (pink sink): H-VER177. The two scalar ports are joined: the tower, completed in the XI chart, is the XI kernel at the zero character, its two-rail carrier is the XI-ENTIRE carrier at $w = 0$, and at the torsion characters it lands on the charge packet. The map accompanies the retained owners and never certifies retention; nothing green consumes it.

Provenance: `H-VER177`.

$$\boxed{\begin{array}{c}\Phi_0^{\mathrm{ch}} = K_\xi^{\mathrm{SCS}}, \qquad \widetilde{\Phi}^{\mathrm{sim}}(u;0) = K_\xi^{\mathrm{SCS}}(u), \qquad \mathcal{L}^{\mathrm{sim}}(\lambda;0) = Q_0^{\mathrm{HG}}(\tfrac{1}{2}+\lambda),\\ e^{-u/2}\,T_u^{\mathrm{HG}}\,e^{u/2} = L_\triangle\,T^{\mathrm{DVJ}}(2L_\triangle\,T^{\mathrm{DVJ}} - 3), \qquad \partial_\rho^{\mathrm{DVJ}} = \tfrac{1}{2}\,\partial_v,\\ \text{scalar map; its FULL40 carriage is H-VER177-A (OPEN).}\end{array}} \tag{14}$$

Open FULL40 aim (sink; no certified target): The scalar map above is complete and proved, and it stays proved off the certified path, which is not the same as not load-bearing. What is not proved is its carriage onto all forty retained labels, `H-VER177-A` (N131A, OPEN): (a.1) the off-lattice same-source DVJ/Jacobi map and the $u \neq 0$ completion carried to the H-VER170 packet; (b) a source intertwiner between the DVJ character derivative and D_u on the retained forty, not only equal differential polynomials; (c) Gate E / A12, a source-faithful map between the carrier zeros, in the H-VER170 Theorem B form of the box above, and DVJ points, the two axes of zero kept distinct. Only that map is open here; Theorem B itself is proved.

Provenance: LOGIC.

$$\boxed{\begin{array}{c}Q_0^{\mathrm{HG}}(\tfrac{1}{2}+\lambda_0^{\mathrm{HG}}) = 0 \;\; (\text{so } J_{40}^{\mathrm{fld}}\,D_Q^{\mathrm{HG}}\,J_{40}^{\mathrm{fld}} = -D_Q^{\mathrm{HG}} \text{ at } s = \tfrac{1}{2}+\lambda_0^{\mathrm{HG}})\\ \lambda_0^{\mathrm{HG}} \;\longleftrightarrow\; \text{DVJ points: open (Gate E / A12),}\\ \text{Gate B: a source intertwiner onto } D_u \text{ on } \widehat{V}_{40},\\ \text{open aim: the scalar map above carried onto all forty labels,}\\ \text{carrier zeros and DVJ points kept distinct; OPEN under H-VER177-A (N131A).}\end{array}} \tag{15}$$

D Boundary and open comparisons

TODO.

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