Full-40 Jacobi–Hypergroup Heat, Mellin and Lerch Source

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Abstract

This paper owns the broad cyclotomic/Jacobi ledger, the complete full-40 Jacobi source, its balanced heat line, the H-VER160 bilateral-entire C8 theta/Mellin owner, and the downstream single-Lerch firewall. H-PAP002 now imports this analytic source package for its DVJ/Jacobi coupling theorems. The retained-ten, trivial-character, classical Mellin and Lerch routes remain typed as reduced, domain-restricted or lossy readouts. The H-VER159 Gamma source carrier is imported from its certified home in H-PAP004. No DVJ coupling theorem and no Riemann-hypothesis claim is made here.

0. Certified full-40 Jacobi–Hypergroup analytic boundary

Reader contract. H-PAP015 is the paper-facing owner of the broad cyclotomic/Jacobi ledger, the complete full-40 source and balanced heat line, the H-VER160 entire-C8 owner, and the downstream single-Lerch firewall. H-PAP002 imports that analytic source package for its genuine DVJ/Jacobi coupling theorems. The rank-ten image, partial Mellin and trivial-character routes remain off the certified path exactly as required by their current owner rulings. H-PAP004 remains the certified home of the H-VER159 Gamma source carrier. H-VER157 and H-VER158 remain in H-PAP002.

Imported sources. The following equations are reproduced only to make the import boundary visible to the reader. Ownership remains with the named Section-0 boxes.

H-PAP008 `front-pap008-h10-source:`

$$\mathcal{B}_{10} = (1, j, \phi_3^1, \phi_3^2, \phi_1, \phi_2, \sigma_1, \sigma_2, \tau_1, \tau_2), \qquad |\mathcal{B}_{10}| = 10.$$

H-PAP001 `front-pap001-archimedean-pi:`

$$\Pi_{\mathrm{src}} := \lim_{n\to\infty} P_n^- = \lim_{n\to\infty} P_n^+, \qquad \Pi_{\mathrm{src}} = \pi.$$

H-PAP001 `front-pap001-analytic-endpoints:`

$$\exp z := \sum_{k=0}^{\infty} \frac{z^k}{k!}, \qquad \sin s := \frac{\exp(is) - \exp(-is)}{2i}.$$

H-PAP004 `front-pap004-hver159-carrier:`

$$\mathcal{G}_{\mathrm{src}}(z) = \int_{-\infty}^{\infty} \mathrm{Exp}_{\mathrm{src}}(zu - \mathrm{Exp}_{\mathrm{src}}(u))\, du, \qquad \mathrm{Re}\, z > 0,$$

$$\mathcal{G}_{\mathrm{src}}(z+1) = z\, \mathcal{G}_{\mathrm{src}}(z), \qquad \mathcal{G}_{\mathrm{src}}(1) = 1.$$

H-PAP004 `front-pap004-gamma-recognition:`

$$\mathcal{G}_{\mathrm{src}}(z) = \int_0^\infty x^{z-1}\exp(-x)\,dx =: \Gamma_{\mathrm{Euler}}(z), \qquad \operatorname{Re} z > 0.$$

Broad Jacobi cyclotomic source: H-VER033, H-VER068, H-VER086.
Certified owners: H-VER033, H-VER068, H-VER086.
Covering provenance: H-VER033, H-VER068, H-VER086.

$$\boxed{\begin{gathered} N \in \{24, 48\}, \\ \mu_N^{\mathrm{prim}} := \left\{\nu \in \mathbb{C} : \nu^N = 1,\ \nu^d \neq 1 \text{ for every } d \mid N,\ 1 \le d < N\right\}, \\ \zeta_N \in \mu_N^{\mathrm{prim}}. \end{gathered}} \tag{1}$$

Retained Jacobi source ledger: H-VER033, H-AUD071-A.
Certified owners: H-VER033, H-AUD071-A.
Covering provenance: H-VER033, H-AUD071-A.

$$\boxed{\begin{gathered} \rho_\triangle(n) := -n-1, \qquad h(n) := 2n+1, \qquad n \in \mathbb{Z}, \\ \chi_{12}(n) := \zeta_{24}^{h(n)}, \qquad \chi_{12}(\rho_\triangle(n)) = \chi_{12}(n)^{-1}, \\ \Omega := 2z\frac{\partial}{\partial z} + 1, \qquad \Omega z^n = h(n) z^n, \\ \vartheta_{00}(z,\tau) := \sum_{n\in\mathbb{Z}} e^{2\pi i n z + \pi i n^2 \tau}, \\ \widehat{\Theta}(X,v) := \sum_{n\in\mathbb{Z}} e^{(2n+1)v} e^{-\pi X (n+1/2)^2}, \\ \tau := iX, \qquad z(X,v) := \frac{v}{\pi i} + \frac{iX}{2}, \\ \widehat{\Theta}(X,v) = e^{v - \pi X/4}\,\vartheta_{00}(z(X,v), iX), \\ (X,v) \longleftrightarrow (iX, z(X,v)). \end{gathered}} \tag{2}$$

Conductor-48 half-character on the ledger index: H-VER033, H-VER092.
Certified owners: H-VER033, H-VER092.
Covering provenance: H-VER033, H-VER092.

$$\boxed{\begin{gathered} \chi_{24}(n) := \zeta_{48}^{h(n)}, \qquad n \in \mathbb{Z}, \qquad \chi_{24}(n)^2 = \chi_{12}(n), \qquad \chi_{24}(n)^{24} = -1, \\ \chi_{24}(n+12) = -\chi_{24}(n), \qquad \chi_{24}(\rho_\triangle(n)) = \chi_{24}(n)^{-1}, \\ \{h(n) \bmod 48 : 0 \le n < 24,\ 3 \nmid h(n)\} = \{a \bmod 48 : \gcd(a,48) = 1\}, \\ n \mapsto n+12 :\ a \mapsto a+24 \text{ (deck)}, \qquad n \mapsto \rho_\triangle(n) :\ a \mapsto -a \text{ (reflection)}. \end{gathered}} \tag{3}$$

Divisor tower of the ledger half-characters: H-VER033, H-VER092.
Certified owners: H-VER033, H-VER092.
Covering provenance: H-VER033, H-VER092.

$$\chi_1(n) := \zeta_2^{h(n)}, \quad \chi_2(n) := \zeta_4^{h(n)}, \quad \chi_3(n) := \zeta_6^{h(n)},$$
$$\chi_4(n) := \zeta_8^{h(n)}, \quad \chi_6(n) := \zeta_{12}^{h(n)}, \quad \chi_8(n) := \zeta_{16}^{h(n)},$$
$$\chi_1(n) = \chi_{24}(n)^{24} = -1, \quad \chi_2(n) = \chi_{24}(n)^{12}, \quad \chi_3(n) = \chi_{24}(n)^8,$$
$$\chi_4(n) = \chi_{24}(n)^6, \quad \chi_6(n) = \chi_{24}(n)^4, \quad \chi_8(n) = \chi_{24}(n)^3, \quad \chi_{12}(n) = \chi_{24}(n)^2,$$
$$\zeta_3^{h(n)} = \chi_3(n)^2, \qquad a = h(n) \bmod 48 \iff \big(h(n) \bmod d\big)_{d|48},$$
$$\chi_8(n+12) = -\chi_8(n), \quad \chi_{24}(n+12) = -\chi_{24}(n), \quad \chi_6(n+12) = \chi_6(n) \text{ (levels dividing 24)},$$
$$\chi_8(\rho_\triangle(n)) = \chi_8(n)^{-1} \text{ and likewise at every level.} \tag{4}$$

Full-40 Jacobi source: H-AUD071-A.
Certified owners: H-AUD071-A.
Covering provenance: H-AUD071-A.

$$\Theta_\ell^{(N)}(z,\tau) := \sum_{k\in\mathbb{Z}} \exp\!\left(\frac{\pi i\tau}{N}(Nk+\ell)^2 + \frac{2\pi i z}{N}(Nk+\ell)\right),$$
$$q := e^{2\pi i\tau}, \qquad \eta(\tau) := q^{1/24}\prod_{m\geq 1}(1-q^m),$$
$$E_{\ell a} := \frac{\delta_{\ell,a} - \delta_{\ell,-a \pmod{12}}}{\sqrt{2}}, \quad \ell \in \mathbb{Z}/12\mathbb{Z}, \quad a \in \{1,2,3,4,5\},$$
$$V_\ell(\tau) := \partial_z \Theta_\ell^{(12)}(z,\tau)\Big|_{z=0}, \qquad V(\tau) := (V_\ell(\tau))_{\ell\in\mathbb{Z}/12\mathbb{Z}},$$
$$\mathbf{v}(\tau) := E^\dagger V(\tau), \qquad \widehat{\mathbf{v}}(\tau) := \frac{\mathbf{v}(\tau)}{\eta(\tau)^3},$$
$$\chi_m^{(8)}(\tau) := \frac{\Theta_m^{(8)}(0,\tau)}{\eta(\tau)}, \qquad m \in \mathbb{Z}/8\mathbb{Z},$$
$$\widehat{V}_{40} := \operatorname{span}\{\widehat{\mathbf{v}}_a \otimes \overline{\chi_m^{(8)}} : a = 1,\dots,5,\ m \in \mathbb{Z}/8\mathbb{Z}\},$$
$$e_{a,m} := e_a^{(5)} \otimes e_m^{(8)}, \qquad a \in \{1,\dots,5\}, \qquad m \in \mathbb{Z}/8\mathbb{Z},$$
$$S_{40}((a,m),(b,m')) := \frac{1}{\sqrt{3}}\sin\frac{\pi ab}{6}\frac{1}{\sqrt{8}}e^{i\pi mm'/4}, \qquad m,m' \in \mathbb{Z}/8\mathbb{Z},$$
$$T_{40}(a,m) := \exp\!\left[\frac{2\pi i}{48}(2a^2 - 3m^2 - 4)\right]. \tag{5}$$

Full-40 balanced heat line: H-AUD071-A, H-VER152.
Certified owners: H-AUD071-A, H-VER152.
Covering provenance: H-AUD071-A, H-VER152.

Full-40 Heat Line, Balanced at $t = 1$

$$\Theta_{40}(t) := \sum_{a=1}^{5}\sum_{m\in\mathbb{Z}/8\mathbb{Z}} \widehat{\mathbf{v}}_a(it^2)\,\overline{\chi_m^{(8)}(it^2)}\,e_{a,m}, \qquad t > 0,$$
$$\Theta_{40}(1/t) = S_{40}\,\Theta_{40}(t) \quad \text{(H-AUD071 S12.3, weight zero, } \tau = it^2 \mapsto -1/\tau = i/t^2\text{)},$$
all 40 coordinates carried: no projection; balanced about the S-fixed point $t = 1$. (6)

Complete entire C8 owner: H-VER160.

Certified owners: H-VER160.
Covering provenance: H-VER160, H-VER152, H-AUD071-A.

H-VER160 Complete Entire C8 Owner

$$
\begin{gathered}
Q_{C_8}^{\mathrm{HG}}(s) := \left(Q_m^{\mathrm{HG}}(s)\right)_{m\in\mathbb{Z}/8\mathbb{Z}}, \qquad Q_m^{\mathrm{HG}}(s) \text{ entire for every } m, \\
Q_0^{\mathrm{HG}}(s) = e_0^T Q_{C_8}^{\mathrm{HG}}(s), \qquad Q_4^{\mathrm{HG}}(s) = e_4^T Q_{C_8}^{\mathrm{HG}}(s), \\
Q_4^{\mathrm{HG}}(s) = (2^s-1)Q_0^{\mathrm{HG}}(s), \\
\left(\widehat{V}_{40}, Q_{C_8}^{\mathrm{HG}}\right) \text{ are retained together,} \\
\text{the C8 view is constructed from the retained full40 source;} \\
\text{no ten-to-C8 replacement or scalar inverse is used.}
\end{gathered}
\tag{7}
$$

H-VER152 and H-VER160 are parallel retained views of the same H-AUD071 source: the source10 heat line, Eq. (9), on one side and full40/entire-C8 on the other. Neither is a replacement for the other. The H-VER152 partial-Mellin section, Eq. (10), is a witness only.

Rank-ten image of the full-40 source: H-AUD071-A owns the recoverable ten-coordinate image; the other thirty source coordinates are not promoted onto the certified path.

Certified owners: H-AUD071-A.
Covering provenance: H-AUD071-A.

$$
\begin{gathered}
\text{full-40 Jacobi source} \xrightarrow[\text{H–AUD071–A}]{\text{rank } 10} \text{retained-ten source,} \\
\text{parity/field-invariant image,} \\
40 = 10 + 30. \\
\Lambda_{20} := \{(a,m) : a \in \{1,\dots,5\},\ m \in \mathbb{Z}/8\mathbb{Z},\ a+m \equiv 1 \pmod 2\}, \\
\mathcal{F}(a,m) := (6-a, m+4), \qquad \mathcal{F}^2 = 1, \qquad \mathcal{F}(a,m) \neq (a,m), \\
\iota_{[a,m]} := \frac{e_{a,m} + e_{6-a,m+4}}{\sqrt{2}}, \qquad [a,m] \in \Lambda_{20}/\langle\mathcal{F}\rangle, \\
\iota : \mathbb{C}^{10} \hookrightarrow \widehat{V}_{40}, \qquad \operatorname{Im}\iota := \operatorname{span}\{\iota_{[a,m]} : [a,m] \in \Lambda_{20}/\langle\mathcal{F}\rangle\}, \\
S_{40}(\operatorname{Im}\iota) = \operatorname{Im}\iota, \qquad T_{40}(\operatorname{Im}\iota) = \operatorname{Im}\iota, \\
\dim \operatorname{Im}\iota = 10, \qquad 10 = 4+4+2, \\
\text{the 30 labels outside } \operatorname{Im}\iota \text{ are not recovered;} \\
\widehat{V}_{40} \text{ stays retained for its full40 consumers.}
\end{gathered}
\tag{8}
$$

H-VER152 / H-AUD071-A retained-ten balanced heat-line packet: H-VER152 evaluates the rank-ten image on the balanced heat line; this reduced packet feeds only readout/witness branches.

Certified owners: H-VER152, H-AUD071-A.
Covering provenance: H-VER152, H-AUD071-A.

H-VER152 Retained-Ten Heat Line, Balanced at $t = 1$

$$\Psi_{10}(\tau) := \big(\psi_a(\tau)\big)_{[a,m]\in\Lambda_{20}/\langle\mathcal{F}\rangle} \in \operatorname{Im}\iota \quad \text{(Eq. (8))},$$
$$\Theta_a(t) := \psi_a(it^2), \qquad \Theta_{\mathrm{vec}}(t) := \big(\Theta_a(t)\big)_{[a,m]\in\Lambda_{20}/\langle\mathcal{F}\rangle}, \qquad t > 0,$$
$$g \in \operatorname{End}(\operatorname{Im}\iota), \qquad (g\Psi_{10})(it^2) = g\Theta_{\mathrm{vec}}(t), \tag{9}$$
$$\Theta_{\mathrm{vec}}(1/t) = \iota^\dagger S_{40}\,\iota\,\Theta_{\mathrm{vec}}(t) \quad \text{(H-AUD071 S12.3, S15.3)},$$

balanced about the S-fixed point $t = 1$; the original ten-owner Ψ_{10} is retained.

$$\text{retained-ten source} \xrightarrow[\text{H–VER152}]{t=1 \text{ balanced heat line}} \text{retained-ten heat packet}.$$

Partial-Mellin witness: The H-VER152 Mellin section is retained for comparison but remains explicitly o-derivation/off-path under the existing owner ruling.

Certified owners: H-VER152.
Covering provenance: H-VER152, H-AUD071-A.

H-VER152 Partial-Mellin Witness (off the critical path)

$$\beta_{\mathrm{M}} \in \mathbb{C}, \qquad \Theta_{\mathrm{vec}} \text{ as in Eq. (9)},$$
$$u_{a,\beta_{\mathrm{M}}}(t) := t^{-i\beta_{\mathrm{M}}} \int_1^t y^{i\beta_{\mathrm{M}}}\Theta_a(y)\frac{dy}{y}, \qquad U_{\beta_{\mathrm{M}}}(t) := \big(u_{a,\beta_{\mathrm{M}}}(t)\big)_{[a,m]\in\Lambda_{20}/\langle\mathcal{F}\rangle},$$
$$L^{(10)}_{\beta_{\mathrm{M}}}\Theta_{\mathrm{vec}} := U_{\beta_{\mathrm{M}}}, \qquad L^{(10)}_{\beta_{\mathrm{M}}}(g\Theta_{\mathrm{vec}}) = gL^{(10)}_{\beta_{\mathrm{M}}}\Theta_{\mathrm{vec}},$$
$$\Theta_a(t) = t^{1-i\beta_{\mathrm{M}}}\frac{d}{dt}\left(t^{i\beta_{\mathrm{M}}}u_{a,\beta_{\mathrm{M}}}(t)\right), \tag{10}$$
$$U_{\beta_{\mathrm{M}}}(1/t) = -\iota^\dagger S_{40}\,\iota\,U_{\beta_{\mathrm{M}}}\big|_{\beta_{\mathrm{M}}\to-\beta_{\mathrm{M}}}(t),$$

based at $t = 1$ (from 0 the integral diverges on this source);
a Mellin witness: it feeds no box on the critical path.

$$\#\{\text{retained heat components}\} = 10,$$
$$\#\{\text{componentwise partial-Mellin witnesses}\} = 10.$$

Trivial-character scalar firewall: H-AUD081 proves that the scalar projection is rank one and loses nine retained coordinates.

Certified owners: H-AUD081.
Covering provenance: H-AUD081, H-VER152, H-AUD071-A, H-VER064, H-VER070.

H-AUD081 Exact Label / Trivial-Character Readout

H-AUD071 orbit order: $(1, \phi_3^1, j, \phi_3^2, \tau_1, \tau_2, \sigma_1, \sigma_2, \phi_1, \phi_2)$,

H-VER064 canonical B_{10} order: $(1, j, \phi_3^1, \phi_3^2, \phi_1, \phi_2, \sigma_1, \sigma_2, \tau_1, \tau_2)$,

$$P_{\mathrm{lab}} := \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad {P_{\mathrm{lab}}}^{-1} = {P_{\mathrm{lab}}}^{T},$$

$$x_{\mathrm{can}} = P_{\mathrm{lab}} x_{\mathrm{orb}},$$

permutation only: no rescaling and no discarded coordinate,

$${p_{\mathrm{triv}}}^{\mathrm{can}} = (1,1,1,1,1,1,1,1,1,1)^T, \qquad {P_{\mathrm{lab}}}^T {p_{\mathrm{triv}}}^{\mathrm{can}} = {p_{\mathrm{triv}}}^{\mathrm{can}},$$

$$\Theta_{\mathrm{triv}}(t) := {p_{\mathrm{triv}}}^T \Theta_{\mathrm{vec}}(t),$$

rank-one scalar readout only; the retained ten-owner is not replaced.

10 retained coordinates $\longrightarrow$ 1 scalar readout + 9 lost directions, $\qquad 10 = 1 + 9.$ (11)

Classical Jacobi recognition: The classical theta presentation is retained as an external recognition of the imported H-VER033/H-AUD071 ledger.

Certified owners: STANDARD.
Covering provenance: STANDARD.

Classical Jacobi Witness (recognition of the H-VER033 / H-AUD071 ledger),

$$\widehat{\Theta}(X, v) = \sum_{n \in \mathbb{Z}} e^{-\pi X (n+a)^2 + 2\pi i (n+a) b}\Big|_{a=1/2,\ b=v/(\pi i)}, \qquad X > 0,$$

$$\widehat{\Theta}(X, v) = X^{-1/2} \sum_{m \in \mathbb{Z}} (-1)^m e^{-(\pi/X)\,(m - v/(\pi i))^2},$$

$$\widehat{\Theta}(X, v + \pi i) = -\widehat{\Theta}(X, v).$$

bilateral Jacobi ledger $\Longrightarrow$

classical characteristic-theta recognition on $X > 0$,

source representation $\cong$ classical theta representation. (12)

Gamma-clean downstream Mellin recognition: The classical Mellin branch consumes H-PAP004's certified H-VER159 carrier; Euler Gamma is downstream recognition, never reverse-ingressed source data.

Certified owners: STANDARD.
Covering provenance: STANDARD, H-VER152, H-VER159.

partial-Mellin witness $\oplus$ H-PAP004 H-VER159 source carrier

$\Longrightarrow$ domain-restricted classical Mellin / Gamma / zeta recognition,

source-side Mellin value = recognized classical value,

recognition $\not\Longrightarrow$ source reconstruction.

Euler Gamma, recognized downstream of the H-VER159 carrier

$$\mathcal{G}_{\mathrm{src}}(z) = \Gamma(z), \qquad \operatorname{Re} z > 0,$$

$$\Gamma(z) := \int_0^\infty t^{z-1} \exp(-t)\, dt, \qquad \operatorname{Re} z > 0,$$

$$\Gamma(z+1) = z\Gamma(z),$$

$$\Gamma(z+2) = z(z+1)\Gamma(z). \tag{13}$$

Classical Mellin Witness

(downstream scalar recognition only)

$$\zeta(s) := \sum_{n\geq 1} n^{-s}, \qquad \operatorname{Re} s > 1,$$

$$\sum_{n\in\mathbb{Z}} \int_0^\infty e^{-\pi X(n+1/2)^2} X^{s/2-1}\, dX = 2\,\pi^{-s/2}\,\Gamma(s/2)\big(2^s-1\big)\zeta(s), \qquad \operatorname{Re} s > 1,$$

this domain-restricted scalar identity is recognition only;

it does not reconstruct the retained ten-owner.

Lossy single-Lerch-rail firewall: One unilateral rail cannot reconstruct the opposing bilateral Jacobi pair.

Certified owners: STANDARD.
Covering provenance: STANDARD.

$$K_+(X,v) := \sum_{n\geq 0} \exp\Big((2n+1)v - \pi X(n+1/2)^2\Big),$$

$$\widehat{\Theta}(X,v) = K_+(X,v) + K_+(X,-v),$$

$\beta_m^j \Phi(\omega_m^j, s, 1/2)$ retained rail,

$\beta_m^{-j} \Phi(\omega_m^{-j}, s, 1/2)$ opposing reflected rail discarded,

discarding the opposing orientation is a lossy downstream readout, LOSSY-DOWNSTREAM-READOU

(14)

Certified H-PAP015 source package: The paper sink reads only the imported H-VER160 entire full-40 owner. Reduced and external-recognition branches above are deliberately not antecedents.

Certified owners: LOGIC.
Covering provenance: LOGIC.

$$\boxed{\begin{array}{c}\mathcal{P}_{015} := (\widehat{V}_{40}, Q^{\mathrm{HG}}_{C_8}, Q^{\mathrm{HG}}_{0}, Q^{\mathrm{HG}}_{4}),\\ \text{imported H-VER160 full-40/C8 entire owner}\\ = \text{EXACT-FULL40-JACOBI-C8-ENTIRE-}\\ \text{MELLIN-SOURCE-PACKAGE.}\end{array}} \tag{15}$$

This checkpoint performs the registered source ownership split. H-PAP002 now imports the full-40 Jacobi source and heat-line package from this paper; H-PAP004 remains unchanged. The finite-PAP008 versus broad-Jacobi cyclotomic distinction remains tracked by BE issue `72f/99c4`. The H10/cyclotomic-to-ledger integer-mode receipt decision remains OPEN under `72f/839`. H-VER157, H-VER158, H-VER150 and H-VER171 remain in the DVJ paper.

1 Retained Jacobi source and Mellin–Lerch witnesses

The Jacobi leg begins from the HG-owned half-character ledger rather than from a remembered theta identity. H-VER033 and H-AUD071 retain the signed integer/half-integer source and only then recognize its standard Jacobi coordinates.

Bilateral half-character source. Equation (2) records the involution $n \mapsto -n-1$, the odd coordinate $h(n) = 2n+1$, and the bilateral series

$$\widehat{\Theta}(X, v) = \sum_{n\in\mathbb{Z}} \exp\big((2n+1)v\big) \exp\big(-\pi X (n+1/2)^2\big). \tag{16}$$

Both reciprocal rails are present. Reflection sends the half-character to its inverse rather than replacing the signed coordinate by an absolute value. This bilateral orientation is part of the source data used later by DVJ/Jacobi cross-checks.

Conductor-48 half-character. Equation (3) reads the same index $h(n)$ at conductor 48, as H-VER033 does for every conductor and H-VER092 does for the signed C48 lift. The square of χ_{24} is the ledger's χ_{12}; the shift $n \mapsto n+12$ negates χ_{24} and $\rho_{\triangle}$ inverts it, so the sixteen primitive conductor-48 labels are indexed by n mod 24. The conductor-24 character keeps neither that sheet nor its deck. No box of this paper consumes χ_{24}, so it stays off the certified path.

Divisor tower. Equation (4) reads the same index at every divisor level of 48: χ_1, χ_2, χ_3, χ_4, χ_6 and χ_8 beside χ_{12} and χ_{24}, each H-VER033's half-character at its own modulus and each a power of χ_{24}. The levels are where different owners read the index: the conductor-24 slot at level 24, the deck that conductor 24 forgets at levels 16 and 48, the fourth-power projection of H-VER092 at level 12, the integer-mode residue modulo four of H-VER158 at level 8, the floor at level 6 and the parity phase at level 4. The shift $n \mapsto n+12$ fixes every level dividing 24 and negates χ_8 and χ_{24}; $\rho_{\triangle}$ inverts every level. No box of this paper consumes the tower, so it stays off the certified path.

Five-vector factor, forty-coordinate carrier, and retained ten embedding. H-AUD071 first forms the five-dimensional Weyl-odd modular factor

$$\mathbf{v} := E^{\dagger} V \in \mathbb{C}^5, \qquad \widehat{\mathbf{v}} := \frac{\mathbf{v}}{\eta^3}. \tag{17}$$

The bold vector $\mathbf{v}$ and its normalized form $\widehat{\mathbf{v}}$ are therefore five-dimensional objects; neither is an alternate name for the retained ten-vector below. H-AUD071 then refines this factor with

the eight-component conjugate character to the forty-coordinate analytic carrier $\widehat{V}_{40}$ and an exact invariant injection

$$\iota : \mathbb{C}^{10} \hookrightarrow \widehat{V}_{40}, \qquad \dim \operatorname{Im} \iota = 10, \qquad \operatorname{Im} \iota \cong O_0^{\mathrm{orb}} \oplus O_1^{\mathrm{orb}} \oplus O_2^{\mathrm{orb}}. \tag{18}$$

The full forty-coordinate source formulae are in Eq. (5); the invariant rank-ten basis and its modular invariance are in Eq. (8). The map is an embedding into an invariant subspace, not a generic forty-to-ten projection. The forty-source and the retained ten coordinates may therefore be carried together without asserting that either is reconstructed from a later scalar readout.

Write the retained analytic source as

$$\Psi_{10}(\tau) = (\psi_a(\tau))_{a \in \mathcal{B}_{10}} \in \operatorname{Im} \iota. \tag{19}$$

This ten-vector, not one scalar theta function, is the Jacobi semantic owner used by H-VER152.

Classical Jacobi recognition. After the source ledger exists, the standard theta function with characteristics recognizes the scalar half-character series. The frozen Section-0 witness gives

$$\boxed{\text{classical Jacobi witness:} \qquad \widehat{\Theta}(X, v) = X^{-1/2} \sum_{m \in \mathbb{Z}} (-1)^m \exp\!\big(-(\pi/X)(m - v/(\pi i))^2\big), \quad X > 0.} \tag{20}$$

This is the classical Jacobi imaginary transformation specialized to the half-shift. The boxed formula is recognition of the already-derived bilateral HG half-character, not an ingress premise. In particular it does not construct Ψ_{10} or $\widehat{V}_{40}$ from a remembered classical theta identity.

A deliberately lossy one-rail witness. For $\operatorname{Re} v < 0$ one may keep only the positive rail

$$K_+(X, v) := \sum_{n \geq 0} \exp\!\big((2n+1)v\big) \exp\!\big(-\pi X (n + 1/2)^2\big), \qquad \widehat{\Theta}(X, v) = K_+(X, v) + K_+(X, -v). \tag{21}$$

The first equality is useful for classical Lerch recognition later, but the single rail has already discarded the opposite orientation. It also carries none of the remaining retained-ten coordinates. The exact ownership is a fork from the common Jacobi source, not an unproved scalar projection from Ψ_{10}:

$$\boxed{\text{common retained Jacobi source} \longrightarrow \begin{cases} \Psi_{10} \subset \widehat{V}_{40}, \\ \widehat{\Theta} \longrightarrow K_+. \end{cases}} \tag{22}$$

No covector $\Psi_{10} \to \widehat{\Theta}$ is asserted here. The source survives because the retained ten-vector and its forty-coordinate parent are kept explicitly beside the lossy scalar views.