Moving Floors and Dual Rails: Hadamard–Cyclotomic Φ_{24} Closure, Signed Golden Traces, and a Renshaw–Padilla Delay Hierarchy

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Working paper. Proposed theorems and biological tests are explicitly marked.

Abstract

This note joins four pieces that had previously been discussed in separate contexts. First, the paired cyclotomic branches $\Phi_3\Phi_6$ and Φ_{12} form a normalized Hadamard pair whose sum and difference expose Φ_8 and the substitution channel x^2. Second, the same two-channel grammar appears in the triangular-number pair (T_+, T_-), whose Hadamard coordinates are (n^2, n). Third, the Golden trace ladder $\mathcal{A}_n = \Phi^{2n} + \Phi^{-2n}$ admits an exact complex quarter-turn refinement $W = \mathrm{diag}(i\Phi, -i\Phi^{-1})$, whose even trace ladder is $(-1)^n\mathcal{A}_n$; scalar trace-squaring then loses this sign information. Fourth, Padilla's self-similar information partition gives $1 - p = \Phi^{-2}$ and hence the same logarithmic unit $\mu_\Phi = 2\log\Phi$ used in the D12RG shell and trace literature.

These facts motivate two testable biological hypotheses. A simple layer-composition model gives a delay fraction $r_L \sim \exp(-L\mu_\Phi) = \Phi^{-2L}$. Its two-stage value lies in the numerical range of weak Renshaw feedback scales reported in the literature, but loop gain and delay ratio are distinct observables and are not identified here. A stronger shell-promotion model uses the triangular exponent $m_n = 1 + T_{n+2} = 4, 7, 11, \ldots$, yielding the visible sequence $1 + \Phi^{m_n} \approx 8, 30, 200, \ldots$. The latter remains a theorem target: it should be derived from the delayed dual-rail hierarchy, not accepted merely because it fits three conspicuous values.

The note also gives an exact affine-torsor interpretation of the "moving floor" language used in FractalCore correspondence: a common translation moves the sum channel but leaves the difference channel invariant. In this sense the floor can move while the relative form is retained.

Two rails trade their names;
twenty-four returns within;
the floor moves, form stays.

1 Scope and status discipline

The purpose is to collect a compact mathematical spine connecting the recent D12RG trace and cyclotomic work with the earlier delay-coupled self-awareness programme and with the recent FractalCore discussion. The note distinguishes three statuses throughout:

1. exact algebraic identities;
2. source-supported biological observations or model results;
3. proposed cross-layer laws that remain to be tested.

The exact trace and cyclotomic statements are downstream of the published or circulated Paper 6.2, Paper 6.4, Paper 6.9, Paper 7.4, and related audit notes [2, 3, 4, 5, 16]. The biological context is downstream of the delay-coupled self-awareness paper and the Paper 6.7 torus-tau bridge [1, 13]. Padilla's information-theoretic result is used only in the form stated by Padilla et al. [6].

No physical admissibility theorem is inferred merely from a matrix identity. In particular, the exact complex trace representation in Section 4 is not, by itself, a proof that a biological or physical system must realize that carrier.

2 The moving floor as a two-rail torsor

The recent FractalCore discussion used the phrase "the floor moves" to describe a base that is not an absolute coordinate origin [10]. A minimal mathematical model for that statement is an affine torsor.

Let $\mathcal{T}$ be a torsor for an additive abelian group G. A pair of rails is represented by $(a, b) \in \mathcal{T} \times \mathcal{T}$. There is no distinguished origin in $\mathcal{T}$, but the difference $a - b \in G$ is well-defined. Under a common translation $c \in G$,

$$(a, b) \longmapsto (a + c, b + c). \tag{1}$$

The differential coordinate is unchanged:

$$(a + c) - (b + c) = a - b. \tag{2}$$

By contrast, after choosing an affine coordinate, the common-mode sum changes by $2c$:

$$(a + c) + (b + c) = (a + b) + 2c. \tag{3}$$

Equations (2) and (3) give a precise form of the phrase:

the floor moves, while the relative form stays.

There is a useful FractalCore-specific refinement. After choosing a local affine chart, write Loker's balanced-null rendering as

$$\varnothing_n = (n, -n). \tag{4}$$

Then the unnormalized Hadamard readout is

$$H_0 \begin{pmatrix} n \\ -n \end{pmatrix} = \begin{pmatrix} 0 \\ 2n \end{pmatrix}. \tag{5}$$

Under a common affine shift c, the same rendered pair becomes

$$H_0 \begin{pmatrix} n + c \\ -n + c \end{pmatrix} = \begin{pmatrix} 2c \\ 2n \end{pmatrix}. \tag{6}$$

In Loker's FractalCore terminology, the two-rail torsor therefore models the balanced null as rendered, or "Rung 1", not the unmarked floor beneath it. The latter is deliberately not represented here by a pair or a number. Equation (6) is the precise fit: the common-mode coordinate records the moved floor, while the differential coordinate retains the rendered relation.

The ordinary two-channel Hadamard map makes this separation explicit. Let

$$H_0 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \qquad D = \frac{1}{2} H_0. \tag{7}$$

The first output is the common-mode sum; the second is the relative or difference mode. Thus the torsor reading is not a metaphor added on top of the algebra: it is the affine geometry naturally associated with the same sum–difference split.

Proposition 1 (Torsor/Hadamard floor invariance)**.** *For a diagonal translation of a two-rail affine pair, the difference Hadamard coordinate is translation-invariant, while the sum coordinate records the displacement of the chosen floor.*

Proof. Apply H_0 to the translated pair in (1). The lower component is exactly (2); the upper component is (3). □

3 Hadamard-in-Hadamard cyclotomic closure

The local signed cyclotomic pair is

$$\Phi_3(x) = x^2 + x + 1, \qquad \Phi_6(x) = x^2 - x + 1. \tag{8}$$

Their product and the level-12 branch are

$$\Phi_3(x)\Phi_6(x) = x^4 + x^2 + 1, \qquad \Phi_{12}(x) = x^4 - x^2 + 1. \tag{9}$$

Applying the normalized Hadamard D from (7) gives the exact pair

$$D\begin{pmatrix}\Phi_3(x)\Phi_6(x)\\ \Phi_{12}(x)\end{pmatrix} = \begin{pmatrix}\Phi_8(x)\\ x^2\end{pmatrix} = \begin{pmatrix}x^4+1\\ x^2\end{pmatrix}. \tag{10}$$

This sum–difference closure is recorded explicitly in the Paper 6.10 inhibitory sum–difference note, where the x^2 output is interpreted as an internal substitution channel rather than merely as negative gain [14].

The triangular dual rail has the same unnormalized Hadamard form. Define

$$T_+(n) = \frac{n(n+1)}{2}, \qquad T_-(n) = \frac{n(n-1)}{2}. \tag{11}$$

Then

$$H_0\begin{pmatrix}T_+(n)\\ T_-(n)\end{pmatrix} = \begin{pmatrix}n^2\\ n\end{pmatrix}. \tag{12}$$

Thus the cyclotomic and triangular packets share a concrete two-rail grammar: a common-mode closure channel and a relative channel that exposes the next substitution coordinate.

3.1 Three direct routes back to Φ_{24}

The first route begins with

$$\Phi_8(w^3) = w^{12} + 1 = \Phi_8(w)\Phi_{24}(w). \tag{13}$$

The second uses the level-12 square lift:

$$\Phi_{12}(z^2) = z^8 - z^4 + 1 = \Phi_{24}(z). \tag{14}$$

Because Φ_{12} is even,

$$\Phi_{12}(+z^2) = \Phi_{12}(-z^2) = \Phi_{24}(z). \tag{15}$$

The third route is

$$\Phi_6(z^4) = z^8 - z^4 + 1 = \Phi_{24}(z). \tag{16}$$

The companion identity

$$\Phi_3(z^4) = \Phi_3(z)\Phi_6(z)\Phi_{12}(z) \tag{17}$$

shows why

$$\Phi_3(z^4)\Phi_6(z^4) = \Phi_3(z)\Phi_6(z)\Phi_{12}(z)\Phi_{24}(z). \tag{18}$$

Thus the earlier shorthand $\Phi_3(z^4)\Phi_6(z^4) = \Phi_{24}(z)$ is too small: the exact factorization contains the complete lower-order packet as well as the Φ_{24} factor.

The three index routes are therefore

$$8 \xrightarrow{\times 3} 24, \qquad 12 \xrightarrow{\times 2} 24, \qquad 6 \xrightarrow{\times 4} 24. \tag{19}$$

Three roads fold to one;
signed squares return the clock;
twenty-four remains.

4 Golden Dickson traces and the quarter-turn subgenerator

The Golden Section is

$$\Phi = \frac{1+\sqrt{5}}{2}. \tag{20}$$

The D12RG logarithmic shell scale is

$$\mu_\Phi = 2\log\Phi. \tag{21}$$

Paper 6.2 and Paper 6.9 use the determinant-one reciprocal trace ladder

$$\mathcal{A}_n = \Phi^{2n} + \Phi^{-2n} = e^{n\mu_\Phi} + e^{-n\mu_\Phi} = 2\cosh(n\mu_\Phi). \tag{22}$$

Equivalently,

$$\mathcal{A}_n = D_n(3,1) = 2T_n(3/2), \tag{23}$$

and

$$\mathcal{A}_{n+1} = 3\mathcal{A}_n - \mathcal{A}_{n-1}, \qquad \mathcal{A}_0 = 2, \quad \mathcal{A}_1 = 3. \tag{24}$$

Hence

$$\mathcal{A}_n = 2, 3, 7, 18, 47, \ldots. \tag{25}$$

Paper 6.9 proves the universal determinant-one trace-square law [4]:

$$\mathcal{A}_n^2 = \mathcal{A}_{2n} + 2. \tag{26}$$

The October 2026 Golden/D48 audit records an exact lower complex trace representation [16]. Let

$$w = i\Phi, \qquad W = \operatorname{diag}(w, w^{-1}) = \operatorname{diag}(i\Phi, -i\Phi^{-1}). \tag{27}$$

Then

$$\det W = 1, \qquad \operatorname{tr} W = i, \qquad \operatorname{tr} W^2 = -3. \tag{28}$$

Define

$$\mathcal{S}_n := \operatorname{tr}(W^n). \tag{29}$$

Cayley–Hamilton gives

$$\mathcal{S}_{n+1} = i\mathcal{S}_n - \mathcal{S}_{n-1}, \qquad \mathcal{S}_0 = 2, \quad \mathcal{S}_1 = i. \tag{30}$$

The even subsequence is

$$\boxed{\mathcal{S}_{2n} = (-1)^n \mathcal{A}_n.} \tag{31}$$

Thus

$$\mathcal{S}_{2n} = 2, -3, 7, -18, 47, \ldots. \tag{32}$$

The sign is not a numerical nuisance. It is a retained orientation bit below the positive trace tower. The exact matrix statement is

$$W^2 = -\operatorname{diag}(\Phi^2, \Phi^{-2}). \tag{33}$$

The sign disappears only after a sign-blind scalar projection such as squaring the even trace:

$$\mathcal{S}_{2n}^2 = \mathcal{A}_n^2 = \mathcal{A}_{2n} + 2. \tag{34}$$

Equation (34) therefore identifies an explicit $\mathbb{Z}_2$ information loss: $+\mathcal{A}_n$ and $-\mathcal{A}_n$ have the same trace-square readout.

The logarithmic coordinate makes the phase origin of the sign visible:

$$\log(i\Phi^{\pm 1}) = \frac{i\pi}{2} \pm \frac{\mu_\Phi}{2} \pmod{2\pi i}. \tag{35}$$

After one square,

$$\log(-\Phi^{\pm 2}) = i\pi \pm \mu_\Phi \pmod{2\pi i}. \tag{36}$$

Thus the quarter-turn phase and the real golden displacement are already present before the positive Dickson trace is formed.

Proposition 2 (Signed quarter-turn trace refinement)**.** *The complex determinant-one carrier in* (27) *is an exact refinement of the positive Golden trace tower in the sense of* (31)*. The scalar trace-square map in* (34) *forgets the parity sign.*

The proposition is algebraic. It does not assert physical selection of W; that remains an admissibility question.

Quarter-turn keeps sign;
squared traces forget the path;
phase waits underneath.

5 Padilla's partition and the same logarithmic unit

Padilla et al. define the balance function [6]

$$f(p) = -(1-p)\ln(1-p) + \ln p. \tag{37}$$

The function in (37) is strictly concave and has its own maximum near

$$p_* \approx 0.882. \tag{38}$$

The Golden partition is a separate statement. It comes from the self-similarity condition

$$p : (1-p) = 1 : p. \tag{39}$$

Equation (39) gives

$$p = \Phi^{-1}, \qquad 1 - p = \Phi^{-2}. \tag{40}$$

Taking the negative logarithm of the residual channel gives

$$-\log(1-p) = 2\log\Phi = \mu_\Phi. \tag{41}$$

Equation (41) is the clean algebraic bridge. It does not identify the maximum p_* in (38) with the Golden point in (40). Rather, Padilla supplies both an empirical/theoretical balance maximum and an independent self-similarity partition; the latter generates exactly the logarithmic unit used in (21).

Proposition 3 (Padilla–Golden logarithmic alignment)**.** *Under Padilla's self-similarity condition* (39)*, the residual information fraction has logarithmic magnitude* μ_Φ *as defined in* (21)*.*

Proof. Use (40) and take $-\log$. □

6 Renshaw delay, PID recurrence, and multilayer suppression

The self-awareness paper models the Renshaw circuit as a delayed neuron–interneuron return and develops the dimensionless delay fraction $r = \tau_{\text{loop}}/T$ as a comparison variable across recurrent biological loops [1]. Williams and Baker independently show that Renshaw recurrent inhibition acts as a frequency-selective neural filter and generates loop-dependent oscillatory effects [7].

The later Paper 6.7 proof work identifies an admissible normalized PID/Renshaw branch with the recurrence

$$F_{t+1} = 3F_t - F_{t-1}, \tag{42}$$

and records that this biological/PID route and the independent Q_3/A_2 spectral route converge on the same reciprocal Golden companion carrier [13].

Equation (42) is already the same second-order trace recurrence as (24). What remains open is the stronger quantitative claim connecting anatomical or functional depth to the logarithmic suppression exponent.

6.1 A simple layer-composition hypothesis

Assume, as a first model, that each independently resolved inhibitory stage contributes one residual factor

$$e^{-\mu_\Phi} = \Phi^{-2}. \tag{43}$$

For L serial or effectively composed stages this gives

$$\boxed{r_L \sim e^{-L\mu_\Phi} = \Phi^{-2L}.} \tag{44}$$

This is a proposed biological law, not a theorem of the source papers. It has, however, an immediate numerical consequence:

Table 1: Simple layer-composition suppression from (44).

L	Φ^{-2L}	interpretation
1	0.381966	single residual unit
2	0.145898	two-stage Renshaw-scale candidate
3	0.055728	intermediate recurrent depth
4	0.021286	sharper hierarchical suppression
5	0.008131	deep-loop candidate

The $L = 2$ value is numerically close to weak Renshaw feedback scales reported outside the project. Williams and Baker obtain a recurrent feedback loop gain $G = 0.12$ in their biophysical model, while Maltenfort, Heckman and Rymer report a mean normalized loop gain near 0.10 with frequency-dependent ranges around that value [7, 8]. Edgley, Williams and Baker independently measure a recurrent-inhibition delay of 4.3 ms, with a 95% confidence interval of 4.1–4.6 ms, in primate upper-limb circuits [9].

These measurements do not directly validate the $L = 2$ row. Loop gain G is not the same observable as the delay fraction $r = \tau_{\text{loop}}/T$, and a delay alone does not define r until the corresponding period T is fixed independently. Moreover, Table 1 displays five candidate values of L. A valid test must assign L from anatomical or functional structure before inspecting r; selecting the closest row afterwards would not count as evidence.

The later Paper 8 analysis makes this caution explicit by introducing an effective exponent. With the convention fixed by $\mu_\Phi = 2\log\Phi$ and (44), define

$$N_{\text{eff}} := -\frac{\log r}{\mu_\Phi} = -\frac{1}{2}\log_\Phi(r), \qquad r \approx \Phi^{-2N_{\text{eff}}}. \tag{45}$$

Then the simple layer model (44) has $N_{\text{eff}} = L$ exactly. The Paper 8 source distinguishes local motif depth from embedding depth [15], so even with this normalization N_{eff} should not be silently identified with one raw neuron count or one anatomical-layer count.

Conjecture 4 (Layered inhibitory suppression)**.** *For a family of recurrent excitatory–inhibitory loops whose stages are comparable under a common normalization, the dimensionless delay or residual fraction is approximately multiplicative in the Padilla/D12RG logarithmic unit:*

$$r \sim \exp(-N_{\text{eff}}\mu_\Phi). \tag{46}$$

The effective exponent should increase with independently defined functional or embedding depth.

7 Triangular exponent promotion

The simple law (44) uses a linear exponent in the number of effective stages. A stronger pattern appeared in the later Paper 8 exploration. Define

$$m_n = 1 + T_{n+2} = 1 + \frac{(n+2)(n+3)}{2}. \tag{47}$$

The first values are

$$m_0 = 4, \qquad m_1 = 7, \qquad m_2 = 11. \tag{48}$$

The corresponding visible ratio candidate is

$$R_n \approx 1 + \Phi^{m_n}. \tag{49}$$

Numerically,

$$R_0 \approx 7.854102, \qquad R_1 \approx 30.034442, \qquad R_2 \approx 200.005025. \tag{50}$$

The sequence in (50) is the origin of the project's recurring shorthand $8:1$, $30:1$, $200:1$. The Paper 8 note correctly marks (49) as the current best candidate among several explored lift models, while leaving its DDE origin open [15].

The relevance of (12) is now sharper. The same triangular pair that produces (n^2, n) under Hadamard readout may also supply the cumulative exponent bookkeeping needed to turn a linear per-stage Golden suppression into the promoted sequence $4, 7, 11, \ldots$.

Conjecture 5 (Triangular promotion theorem target)**.** *A nested delayed dual-rail hierarchy with stage-dependent comparison and accumulation admits a coarse-grained exponent whose increments are* $3, 4, 5, \ldots$*, so that the accumulated exponent is* $m_n = 1 + T_{n+2}$ *as in* (47)*.*

A proof must derive the increments from the DDE or carrier composition. Fitting (50) after the fact is not sufficient.

8 Proposed theorem and falsification programme

The current material separates naturally into exact algebra and open cross-layer tests.

8.1 Exact theorem package

The following items are already elementary or source-proved:

E1. torsor difference invariance, Proposition 1;

E2. the cyclotomic Hadamard split (10);

E3. the three Φ_{24} routes (13)–(16);

E4. the Golden trace recurrence (24) and square law (26);

E5. the signed quarter-turn trace refinement (31);

E6. the sign loss under scalar trace-square projection (34);

E7. the Padilla logarithmic alignment (41).

8.2 Open theorem targets

The next mathematical targets are:

T1. derive (46) from an explicit composition of normalized delayed controllers;

T2. determine when N_{eff} equals literal stage count and when it instead depends on motif and embedding depth;

T3. derive the triangular exponent law (47) from the dual-rail DDE hierarchy;

T4. determine whether the i-phase sign retained by W supplies the missing orientation state needed by that derivation;

T5. classify which parts of the Φ_{24} multi-route closure remain invariant under admissible operator substitution.

8.3 A narrow biological test

The October 2026 correspondence with Arturo Salazar Chon emphasized a useful methodological boundary: specify one observable, one expected relationship, one comparator, and one result that would count against the hypothesis before adding the result to an existing analysis.

Proposed test 6 (Delay-ratio exponent test)**.** *For a predeclared family of recurrent circuits, estimate*

$$r = \frac{\tau_{\mathrm{loop}}}{T}, \tag{51}$$

where τ_{loop} is an independently defined loop delay and T is an independently defined coherent or recurrence period. Compute N_{eff} using (45).

The recurrent depth or stage count must be assigned before r is inspected. The hypothesis predicts increasing N_{eff} with that independently defined depth. A failure to obtain stable clustering across held-out data, or a systematic ordering opposite to the preregistered depth ordering, counts against the hypothesis.

This test does not identify seizure dynamics, H_1 persistence, Ricci curvature, or any other observable with consciousness. It is a separate quantitative test of a delay-suppression law.

9 Relation to Tom Loker's "moving floor"

The recent correspondence with Tom Loker was motivated by the fact that a two-rail construction can retain a relational coordinate even when the chosen common reference changes. Section 2 supplies the exact affine statement.

The relation to the FractalCore "floor" should therefore be stated carefully. The torsor theorem does not prove the cosmological or ontological claims of the FractalCore floor paper [10]. It does supply a concrete mathematical model of one phrase used in that discussion. In the terminology of Section 2, the rendered balanced null is the first two-rail object; the unmarked floor itself remains prior to that rendering.

Earlier ICFA architecture already contained literal layerwise subtraction and a cross-layer role exchange or swap. Loker's August 2026 source audit described the correspondence as structural rather than literally the present 2×2 Hadamard matrix [12]. The broader ICFA hierarchy is documented by Loker and Lowy [11]. The present note therefore credits ICFA with the earlier subtract/swap primitive, while the explicit Hadamard dual-rail algebra and the requirement to preserve the relative channel are the later identification made here.

The mathematical floor statement is

$$\text{common origin moves} \quad \Longrightarrow \quad \text{difference form can remain invariant.} \tag{52}$$

This is precisely what the Hadamard decomposition does: the common-mode rail records the floor displacement, while the differential rail records the relation that survives it.

There is a second connection. In the Golden trace system, passing from W to the sign-blind scalar square similarly discards an orientation coordinate while retaining a positive invariant. The two cases should not be identified, but they have the same information-theoretic warning:

do not mistake an invariant readout for the whole carrier.

Sum moves with the floor;
difference keeps what was meant;
two keep the one true.
— Tom Loker, correspondence, 27 October 2026

10 Discussion

The combined structure can be summarized as the following chain:

$$\begin{aligned}&\text{dual rail / torsor} \longrightarrow \text{Hadamard sum–difference} \longrightarrow \Phi_{24} \text{ route closure}\\ &\longrightarrow \mu_\Phi = 2\log\Phi \longrightarrow \Phi^{\pm 2n} \text{ reciprocal trace} \longrightarrow W \text{ signed refinement}\\ &\longrightarrow \text{Padilla residual log unit} \longrightarrow \text{Renshaw delay hypothesis} \longrightarrow \text{multilayer promotion.}\end{aligned} \tag{53}$$

Several arrows in (53) are exact algebra. The final biological arrows are not. This distinction is a strength rather than a weakness: the algebra now supplies precise observables and failure conditions instead of a loose analogy.

The most important new conceptual point is the location of sign and phase information. The positive Dickson trace $\mathcal{A}_n = \Phi^{2n} + \Phi^{-2n}$ is not the lowest available carrier. The quarter-turn matrix W in (27) retains an imaginary orientation, and its even trace ladder alternates sign. The scalar square (34) then proves exactly where one bit of this orientation becomes invisible.

This may matter for the biological hierarchy. Delayed inhibition is a phase operation as well as an amplitude operation. If the correct multilayer derivation depends on retained orientation, then attempting to derive the biological law only from the positive trace tower may be structurally incapable of recovering the required stage information. That is a concrete reason to test the signed carrier rather than treating it as ornamental complex notation.

11 Conclusion

The WIP result is not a single grand theorem. It is a compact interface between several exact structures and two falsifiable extensions.

The exact side contains:

$$\boxed{\begin{gathered}D(\Phi_3\Phi_6, \Phi_{12})^T = (\Phi_8, x^2)^T,\\ H_0(T_+, T_-)^T = (n^2, n)^T,\\ \mu_\Phi = 2\log\Phi, \qquad \mathcal{A}_n = \Phi^{2n} + \Phi^{-2n},\\ \mathcal{S}_{2n} = (-1)^n\mathcal{A}_n, \qquad \mathcal{S}_{2n}^2 = \mathcal{A}_{2n} + 2,\\ -\log(1-p) = \mu_\Phi \quad \text{at the Padilla Golden partition.}\end{gathered}} \tag{54}$$

The proposed side asks whether these same logarithmic and phase coordinates govern increasingly sharp inhibitory return as recurrent neural depth increases, and whether the triangular dual rail derives the observed candidate promotion sequence $4, 7, 11, \ldots$.

That programme is sufficiently precise to fail. It is therefore sufficiently precise to test.

Delay folds through depth;
each return narrows the gate;
golden rails breathe slow.

A Condensed correspondence note to Tom Loker

The core message from the recent FractalCore exchange can be stated in a short form.

The two rails are not merely two copies of the same value. Their sum and difference carry different information. In the triangular basis, (12) reads the pair (T_+, T_-) as (n^2, n). In the cyclotomic basis, (10) reads $(\Phi_3\Phi_6, \Phi_{12})$ as (Φ_8, x^2).

The output Φ_8 then contains another route to Φ_{24} through (13), while the signed square lifts of Φ_{12} both return to Φ_{24} through (15). Thus a very small add–subtract–substitute grammar carries a surprisingly deep hierarchy.

The torsor observation adds the moving-floor interpretation. A common translation changes the common-mode coordinate but not the relative coordinate. The floor can therefore move without destroying the form that is encoded relationally.

The safe operator statement is similarly narrow. Because the identities are single-variable polynomial identities, they may be evaluated at one matrix or operator X in a unital associative algebra without creating an ordering ambiguity. For example,

$$\Phi_{12}(X^2) = \Phi_{24}(X). \tag{55}$$

This algebraic extension does not imply that every such operator is physically selected. Physical selection remains a separate admissibility question.

Two rails trade their names;
twenty-four returns within;
the floor moves, form stays.

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