Bilateral Gaussian Zero-Mode Retention and an Entire Quarter-Characteristic Riemann Carrier: The Failed Routes, Recursive Half-Shift, and Exact Equivalence

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27th September 2026

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Contact: lkcl@lkcl.net Released: 27th September 2026.
ORCID: 0009-0006-8300-6257 0000-0001-9090-5177
DOI: 10.13140/RG.2.2.20814.22089
https://researchgate.net/publication/414857196

Abstract

We construct an exact bilateral, zero-inclusive Gaussian formulation of a Riemann-hypothesis (RH) equivalent, preserving the distinction between the original lattice zero mode and its Fourier-dual coefficient. The completed scalar carrier is entire in the complex variable s, and hence entire in $z \in \mathbb{C}$ under the quarter-turn $s = \frac{1}{2} + iz$. A recursive half-shift yields four indexed Gaussian characteristics; their invertible packet retains the quarter-odd coordinate that the scalar projection loses. The quarter kernel has a one-tail logarithmic Mellin transform, and Gaussian completion identifies its entire scalar carrier with a known multiplier of completed Riemann $\xi(s)$. The complete plus/minus half-rail pair provides distinct two-tail and one-tail completions and exact identities $Q^{+}_{2/4} = 2^s\xi(s)$, $Q^{-}_{2/4} = (2 - 2^s)\xi(s)$, and $Q^{+}_{2/4} + Q^{-}_{2/4} = 2\xi(s)$. The original and reflected carriers are distinguished throughout; reflected right-half-plane zero-freeness does not establish RH. We retain the erroneous positive-index and post-hoc zero-mode routes as reviewable provenance. The quarter multiplier is nonzero within the open critical strip, giving an exact equivalent zero-free condition, not verification of that condition. A scalar readout does not establish a source-preserving hypergroup/De Vries Jacobi Jordan Order $N = 3$ nilpotent tower (DVJ) intertwiner. Revision of 13 October 2026: the carrier is a scalar readout of the indexed four-rail packet, which cancels the odd quarter channel, so its exact RH equivalence is downgraded to an audit result. The certified content is the indexed packet; Section 11 explains why, including why the standard scalar formulation of RH is itself information-lossy.

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Reader contract. A bilateral lattice term at index zero, the Fourier-dual index zero and the logarithmic origin are different objects. Equivalence of zero conditions is not a proof of zero-freeness. Classical inputs are cited where introduced; all other displayed identities require the algebra and convergence checks in the TODO body.

Revision of 13 October 2026 (downgrade). The scalar carrier is a lossy readout of the indexed four-rail packet, so its exact RH equivalence is no longer the certified sink. Section 0 now certifies the indexed packet only, (40); every box of the scalar chain stays displayed, off the certified path, with its reason. See Section 11.

Imported sources. The following equations are owned by another paper and imported, not redrawn; each item names its owner box.

- H-PAP001 [1], box front-pap001-archimedean-pi (owner H-VER077): $\Pi_{\mathrm{src}} := \lim_{n\to\infty} P_n^- = \lim_{n\to\infty} P_n^+$ and $\Pi_{\mathrm{src}} = \pi$. Ordinary π enters only through this endpoint recognition.

- H-PAP001 [1], box front-pap001-analytic-endpoints (owner H-VER155): $\exp z := \sum_{k=0}^{\infty} z^k/k!$.

Provenance: logic.

$$\boxed{\begin{gathered} x > 0, \quad a \in \mathbb{R}, \quad n, k \in \mathbb{Z}, \quad s \in \mathbb{C}, \\ p_a(n,x) := e^{-\pi(n+a)^2 x}, \qquad p_0(n,x) := p_a(n,x)\big|_{a=0}, \\ p_0(0,x) = 1. \end{gathered}} \tag{1}$$

The original index-zero term is present, not assigned a value after subtraction. See Section 1 and [7].

Audit box, off the certified path: logic, standard.

$$\boxed{\begin{gathered} \Theta_a(x) := \sum_{n\in\mathbb{Z}} p_a(n,x), \qquad \Theta_0(x) := \Theta_a(x)\big|_{a=0} = \sum_{n\in\mathbb{Z}} p_0(n,x), \\ \text{the bilateral series of the terms of (1); its } n=0 \text{ term is } p_0(0,x) = 1. \end{gathered}} \tag{2}$$

A readout of the indexed terms, kept for the classical comparisons that follow. See Section 11.

Audit box, off the certified path: standard.

$$\boxed{\begin{gathered} \xi(s) := \tfrac{1}{2} s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s), \\ \xi(s) = \xi(1-s), \quad \xi : \mathbb{C} \to \mathbb{C} \text{ entire.} \end{gathered}} \tag{3}$$

The standard completed xi function is not the original lattice zero mode. See Section 2 and [6].

Audit box, off the certified path: standard.

$$\boxed{\begin{gathered} \omega(x) := \sum_{n=1}^{\infty} e^{-\pi n^2 x} = \tfrac{1}{2}(\Theta_0(x) - 1), \\ \xi(s) = \tfrac{1}{2} + \frac{s(s-1)}{4} \int_1^{\infty} (\Theta_0(x) - 1) \\ \cdot \left(x^{s/2} + x^{(1-s)/2} \right) \frac{dx}{x}, \\ \xi(0) = \xi(1) = \tfrac{1}{2}. \end{gathered}} \tag{4}$$

Exact completion, but the displayed omega subtracts the original central mode; this is not a DVJ zero-mode witness. See Section 3 and [4, 5].

Firewall: logic.

$$\boxed{\begin{gathered}\int_0^1 x^{s/2-1}\,dx \text{ converges iff } \Re s > 0,\\ \int_1^\infty x^{s/2-1}\,dx \text{ converges iff } \Re s < 0,\\ \int_0^\infty x^{s/2-1}\,dx \text{ has no ordinary convergence domain.}\end{gathered}} \tag{5}$$

This is the obstacle for an isolated constant Gaussian mode, not an impossibility theorem for RH. See Sections 3 and 6.

Firewall: logic.

$$\boxed{\begin{gathered}\mathcal{L}(w_q) := \log\left[2\xi\left(\frac{-w_q}{1-w_q}\right)\right] = \sum_{n=1}^{\infty}\frac{\lambda_n}{n}w_q^n,\\ \mathcal{L}(0) = 0 \quad \not\Rightarrow \quad \text{original lattice-index-zero retained.}\end{gathered}} \tag{6}$$

RETRACTED INFERENCE: Li's valid criterion does not establish a Gaussian n=0 source coordinate. See Section 2 and [12, 21].

Firewall: logic, standard.

$$\boxed{\begin{gathered}8\xi(\tfrac{1}{2}+z) = \sum_{\nu=0}^{\infty}\frac{\gamma_\nu}{\nu!}z^{2\nu},\\ \gamma_\nu = \frac{8\nu!}{(2\nu)!}\xi^{(2\nu)}(\tfrac{1}{2}),\\ \gamma_\nu|_{\nu=0} = 8\xi(\tfrac{1}{2}) \neq 0,\\ J^{d,\nu}(X) := \sum_{j=0}^{d}\binom{d}{j}\gamma_{\nu+j}X^j,\\ \text{RH} \iff \text{all } J^{d,\nu} \text{ are hyperbolic } (d \geq 1, \nu \geq 0).\end{gathered}} \tag{7}$$

The Taylor index is not the Gaussian index. The coefficient convention includes index zero; the criterion does not establish retention of the original lattice mode. See Section 2 and [11, 21].

Firewall: logic.

$$\boxed{\begin{array}{c} K_n(s) := \tfrac{1}{2}\delta_{n,0} + \dfrac{s(s-1)}{4}\displaystyle\int_1^\infty \\ \left(e^{-\pi n^2 x} - \delta_{n,0}\right)\left(x^{s/2} + x^{(1-s)/2}\right)\dfrac{dx}{x}, \\ K_n\big|_{n=0} = \tfrac{1}{2} \quad \not\Rightarrow \quad \text{unsubtracted source-mode transform.} \end{array}} \tag{8}$$

RETRACTED source-identification: the K coefficient was introduced after the theta subtraction. See Section 3 and [21].

Firewall: logic.

$$\boxed{\begin{array}{c} F_{\text{pair}}(s) := (2^s - 1)(2^{1-s} - 1)\xi(s), \\ F_{\text{pair}}(0) = 0 \quad \text{although} \quad \xi(0) = \tfrac{1}{2}. \end{array}} \tag{9}$$

Exact factor identity, rejected as the alpha-zero normalization because it creates an endpoint zero. See Section 2 and [21].

Provenance: logic.

$$\boxed{\begin{array}{c} p_{2/4}(n,x) := p_a(n,x)\big|_{a=2/4}\,, \qquad p_{2/4}(n,x) = e^{-\pi(n+1/2)^2 x}, \\ p_0(2n+2a,x) = p_a(n,4x), \quad a \in \{0,2/4\}: \quad p_0(2n+1,x) = p_{2/4}(n,4x), \\ (n,a) \mapsto 2n+2a: \ \mathbb{Z}\times\{0,2/4\} \to \mathbb{Z} \text{ is a bijection.} \end{array}} \tag{10}$$

First split n=2m and n=2m+1, with all m in Z. See Section 4 and [7].

Provenance: logic.

$$\boxed{\begin{array}{c} p_{1/4}, p_{3/4} := p_a\big|_{a=1/4}\,, p_a\big|_{a=3/4}\,, \\ p_{2/4}(2n,x) = p_{1/4}(n,4x), \qquad p_{2/4}(2n+1,x) = p_{3/4}(n,4x), \\ p_0(4n+4a,x) = p_a(n,16x), \quad a \in \{0,1/4,2/4,3/4\}, \\ (n,a) \mapsto 4n+4a: \ \mathbb{Z}\times\{0,1/4,2/4,3/4\} \to \mathbb{Z} \text{ is a bijection}, \\ p_{1/4}(0,x) = e^{-\pi x/16}, \qquad p_{3/4}(0,x) = e^{-9\pi x/16}. \end{array}} \tag{11}$$

Each term of the original index goes to exactly one term of one of the four cosets, and back; nothing is summed. The series built from these terms are in (12), off the certified path. See Section 4.

Audit box, off the certified path: logic, standard.

$$\boxed{\begin{gathered}\Theta_a(x)=\Theta_{a/2}(4x)+\Theta_{(a+1)/2}(4x),\qquad \Theta_0(x)=\Theta_0(4x)+\Theta_{1/2}(4x),\\ \Theta_0(x)=\Theta_0(16x)+\Theta_{1/2}(16x)+\Theta_{1/4}(16x)+\Theta_{3/4}(16x),\\ \text{the series over } n \text{ of the per-index identities (10) and (11).}\end{gathered}} \tag{12}$$

The quarter and three-quarter full theta functions agree after reindexing $n \mapsto -n-1$, so their difference, the sum of the odd quarter channel of (15), is zero; the coset labels survive only in the indexed packet. See Section 4 and [7].

Provenance: logic.

$$\boxed{\begin{gathered}x=e^u,\quad u\in\mathbb{R},\quad \frac{dx}{x}=du,\\ s=\tfrac{1}{2}+it,\qquad x^{s/2}+x^{(1-s)/2}=2e^{u/4}\cos(tu/2),\\ u=0 \iff x=1 \quad (\text{not } \log 0).\end{gathered}} \tag{13}$$

The quarter exponent in log scale is distinct from the quarter-characteristic index. See Sections 4 and 7.

Audit box, off the certified path: logic.

$$\boxed{\begin{gathered}H_{1/4}(x):=\Theta_0(x)-\tfrac{1}{2}(\Theta_{1/4}(x)+\Theta_{3/4}(x)),\\ \Theta_0|_{n=0}=1,\qquad \lim_{x\to\infty}H_{1/4}(x)=1.\end{gathered}} \tag{14}$$

This new combination is derived here; do not attribute the combination itself to NIST. It is a scalar readout of the indexed packet (revision of 13 October 2026; see Section 11). See Sections 5 and 8.

Provenance: logic.

$$\boxed{\begin{gathered}\text{the four indexed rails of (1)–(11) at each } (n,x):\\ \begin{pmatrix}\mathsf{A}\\ \mathsf{B}\\ \mathsf{E}\\ \mathsf{O}\end{pmatrix}\Bigg|_{(n,x)}=\frac{1}{2}\begin{pmatrix}1&1&0&0\\ 1&-1&0&0\\ 0&0&1&1\\ 0&0&1&-1\end{pmatrix}\begin{pmatrix}p_0\\ p_{2/4}\\ p_{1/4}\\ p_{3/4}\end{pmatrix}\Bigg|_{(n,x)},\\ p_0=\mathsf{A}+\mathsf{B},\quad p_{2/4}=\mathsf{A}-\mathsf{B},\\ p_{1/4}=\mathsf{E}+\mathsf{O},\quad p_{3/4}=\mathsf{E}-\mathsf{O},\end{gathered}} \tag{15}$$

The 3-by-3 inverse covers only the selected triple; this four-coordinate inverse retains the independent half rail and quarter-rail orientation. Its sum over n, which gives the kernel, is in (20) (revision of 13 October 2026). See Section 4.

Provenance: logic.

$$\boxed{\begin{gathered} p_0(-n,x) = p_0(n,x), \quad p_{2/4}(-n-1,x) = p_{2/4}(n,x), \\ p_{1/4}(-n-1,x) = p_{3/4}(n,x), \quad \mathsf{O}(-n-1,x) = -\mathsf{O}(n,x), \\ p_{2/4}(0,x) = e^{-\pi x/4}, \quad \mathsf{O}(0,x) = \tfrac{1}{2}(e^{-\pi x/16} - e^{-9\pi x/16}) \neq 0, \\ \sum_{n\in\mathbb{Z}} \mathsf{O}(n,x) = 0 \quad (n \leftrightarrow -n-1). \end{gathered}} \tag{16}$$

Per-index identities of the packet, before any sum. The last line is why every summed readout loses the odd quarter channel: the sum pairs n with -n-1 and cancels, although no indexed term vanishes. See Sections 4 and 11.

Audit box, off the certified path: logic.

$$\boxed{\begin{gathered} \Theta_{1/4}(x) + \Theta_{3/4}(x) = \Theta_0(x/16) - \Theta_0(x/4), \\ H_{1/4}(x) = \Theta_0(x) + \tfrac{1}{2}\Theta_0(x/4) - \tfrac{1}{2}\Theta_0(x/16). \end{gathered}} \tag{17}$$

The full unshifted theta functions are retained. The individual Mellin transforms of these three terms have no common ordinary convergence strip; do not split the raw integral by fiat. See Sections 4, 6 and 8.

Audit box, off the certified path: logic.

$$\boxed{\begin{gathered} G_{1/4}(n,u) := e^{-\pi n^2 e^u} - \tfrac{1}{2}e^{-\pi(n+1/4)^2 e^u} \\ -\tfrac{1}{2}e^{-\pi(n+3/4)^2 e^u}, \\ H_{1/4}(e^u) = \sum_{n\in\mathbb{Z}} G_{1/4}(n,u), \\ G_{1/4}(0,u) = 1 - \tfrac{1}{2}e^{-\pi e^u/16} - \tfrac{1}{2}e^{-9\pi e^u/16}. \end{gathered}} \tag{18}$$

The sum remains inside the logarithmic integral. This individual n=0 term is not the total kernel. See Sections 5 and 6.

Audit box, off the certified path: logic, standard.

$$\boxed{\begin{aligned} W_{1/4}(k) &:= 1-\cos\left(\frac{\pi k}{2}\right), \\ H_{1/4}(x) &= x^{-1/2}\sum_{k\in\mathbb{Z}} W_{1/4}(k)e^{-\pi k^2/x}, \\ W_{1/4}(0) &= 0, \qquad \lim_{x\to\infty} H_{1/4}(x) = 1. \end{aligned}} \tag{19}$$

The dual k=0 coefficient vanishes by Fourier cancellation; the original n=0 mode is still present. See Section 5 and [8].

Audit box, off the certified path: logic, standard.

$$\boxed{\begin{aligned} K^{\mathrm{idx}}_{1/4}(n,x) &:= p_0 - \mathsf{E} = \mathsf{A}+\mathsf{B}-\mathsf{E} \quad (\text{no } \mathsf{O} \text{ term}), \\ \sum_{n\in\mathbb{Z}} K^{\mathrm{idx}}_{1/4}(n,x) &= H_{1/4}(x) \quad (\text{the scalar readout of the packet}), \\ a = 0,1/4,2/4,3/4: &\quad e^{2\pi i k a} = 1, i^k, (-1)^k, (-i)^k, \\ \sum_{n\in\mathbb{Z}} \mathsf{A}(n,x) &= x^{-1/2}\sum_{k\in\mathbb{Z}} \tfrac{1+(-1)^k}{2} e^{-\pi k^2/x}, \\ \sum_{n\in\mathbb{Z}} \mathsf{B}(n,x) &= x^{-1/2}\sum_{k\in\mathbb{Z}} \tfrac{1-(-1)^k}{2} e^{-\pi k^2/x}, \\ \sum_{n\in\mathbb{Z}} \mathsf{E}(n,x) &= x^{-1/2}\sum_{k\in\mathbb{Z}} \cos(\pi k/2) e^{-\pi k^2/x}, \\ \sum_{n\in\mathbb{Z}} \mathsf{O}(n,x) &= i x^{-1/2}\sum_{k\in\mathbb{Z}} \sin(\pi k/2) e^{-\pi k^2/x} = 0. \end{aligned}} \tag{20}$$

The summed odd channel cancels, but its indexed summands need not vanish; the per-index reflections are (16) (revision of 13 October 2026). The half rail has its own phase and zero. See Sections 4 and 5; [8].

Audit box, off the certified path: logic, standard.

$$\boxed{\begin{aligned} &\sum_{n\in\mathbb{Z}} \mathsf{E}(n,x) = \Theta_{1/4}(x) = \Theta_{3/4}(x) \\ &= \tfrac{1}{2}\Theta_{1/2}(x/4) = \tfrac{1}{2}\left[\Theta_0(x/16) - \Theta_0(x/4)\right], \\ &= x^{-1/2}\sum_{k\in\mathbb{Z}} \cos(\pi k/2) e^{-\pi k^2/x}, \\ &\sum_{n\in\mathbb{Z}} \mathsf{O}(n,x) = 0, \quad \mathsf{O}(0,x) \neq 0, \\ &\cos(0) = 1 \quad (\text{dual } k=0 \text{ coefficient}). \end{aligned}} \tag{21}$$

The half in the full-theta difference is a coefficient, not the value of the E sum. Scalarization loses indexed quarter orientation. See Sections 4 and 5; [8].

Audit box, off the certified path: logic, standard.

$$\begin{gathered}
H^{+}_{2/4}(x) := \sum_{n\in\mathbb{Z}} (p_0 + p_{2/4}) = \Theta_0(x/4), \\
H^{-}_{2/4}(x) := \sum_{n\in\mathbb{Z}} (p_0 - p_{2/4}) = 2\Theta_0(x) - \Theta_0(x/4), \\
p_0, p_{2/4} \text{ in the sums are evaluated at } (n, x), \\
H^{+}_{2/4}(x) = x^{-1/2} \sum_{k\in\mathbb{Z}} (1 + (-1)^k) e^{-\pi k^2/x}, \\
H^{-}_{2/4}(x) = x^{-1/2} \sum_{k\in\mathbb{Z}} (1 - (-1)^k) e^{-\pi k^2/x}, \\
w_0^{+} = 2, \quad w_0^{-} = 0, \qquad \lim_{x\to\infty} H^{+}_{2/4}(x) = \lim_{x\to\infty} H^{-}_{2/4}(x) = 1.
\end{gathered} \tag{22}$$

The plus dual constant survives; the minus dual constant cancels. These are scalar sums of the indexed A/B pair, not recovery of the independent E/O coordinates. See Sections 4 and 5; [8].

Audit box, off the certified path: logic.

$$\begin{gathered}
H_{1/4}(x) = O\Big(x^{-1/2} e^{-\pi/x}\Big) \quad (x \to 0^{+}), \\
H_{1/4}(x) = 1 + O\Big(e^{-\pi x/16}\Big) \quad (x \to \infty), \\
\mathcal{D}_{1/4} := \{s \in \mathbb{C} : \Re s < 0\}.
\end{gathered} \tag{23}$$

Both asymptotics follow from the bilateral Gaussian bounds. See Sections 5 and 6.

Audit box, off the certified path: logic.

$$\begin{gathered}
\mathcal{M}_{1/4}(s) := \int_{-\infty}^{\infty} e^{su/2} \left[\sum_{n\in\mathbb{Z}} G_{1/4}(n, u) \right] du, \\
s \in \mathcal{D}_{1/4}, \quad \mathcal{M}_{1/4} \text{ converges absolutely.}
\end{gathered} \tag{24}$$

Retain the Z-indexed sum as a whole, including its n=0 term. See Sections 6 and 7.

Firewall: logic.

$$\boxed{\begin{array}{c}\mathcal{M}_{1/4}(s) = 2\pi^{-(1-s)/2}\Gamma\left(\dfrac{1-s}{2}\right) \\ \cdot \displaystyle\sum_{k=1}^{\infty} \frac{1-\cos(\pi k/2)}{k^{1-s}}, \\ \text{valid cross-check, but not a bilateral-index} \\ \text{source-preserving presentation.}\end{array}} \tag{25}$$

RETRACTED AS CANONICAL REPRESENTATION: the termwise step reinstates the positive-index formulation the user expressly rejected. See Section 6 and [21].

Provenance: logic.

$$\boxed{\begin{array}{c}\chi(u) := \tfrac{1}{2}(1+\operatorname{erf}(u)), \\ \lim_{u\to-\infty}\chi(u) = 0, \qquad \lim_{u\to+\infty}\chi(u) = 1.\end{array}} \tag{26}$$

This cutoff is bookkeeping for the original constant tail, not an independent value assigned to n=0. See Section 7.

Audit box, off the certified path: logic.

$$\boxed{\begin{array}{c}R_{1/4}(s) := \displaystyle\int_{-\infty}^{\infty} e^{su/2}\left[\sum_{n\in\mathbb{Z}} G_{1/4}(n,u) - \chi(u)\right] du, \\ R_{1/4} : \mathbb{C} \to \mathbb{C} \quad \text{entire}, \\ H_{1/4}(e^u) - \chi(u) = O\left(\dfrac{e^{-u^2}}{1+|u|}\right) \quad (|u| \to \infty).\end{array}} \tag{27}$$

The subtraction acts on the full summed kernel; no individual n=0 term undergoes a divergent Mellin transform. See Section 7.

Audit box, off the certified path: logic.

$$\boxed{\begin{array}{c}\displaystyle\int_{-\infty}^{\infty} e^{su/2}\chi(u)\,du = -\frac{2}{s}e^{s^2/16}, \\ s \in \mathcal{D}_{1/4}, \qquad \mathcal{M}_{1/4}(s) = R_{1/4}(s) - \dfrac{2}{s}e^{s^2/16}.\end{array}} \tag{28}$$

The raw Mellin pole at s=0 is -2/s; the compensator moment is evaluated only where the integral converges. See Section 7.

Audit box, off the certified path: logic.

$$\boxed{\begin{gathered} R^{-}_{2/4}(s) := \int_{\mathbb{R}} e^{su/2}[H^{-}_{2/4}(e^u) - \chi(u)]\,du, \\ C^{+}_{2/4}(u) := \chi(u) + 2e^{-u/2}\chi(-u), \\ R^{+}_{2/4}(s) := \int_{\mathbb{R}} e^{su/2}[H^{+}_{2/4}(e^u) - C^{+}_{2/4}(u)]\,du, \\ R^{-}_{2/4}, R^{+}_{2/4} : \mathbb{C} \to \mathbb{C} \quad \text{entire}, \\ \int_{\mathbb{R}} e^{su/2} H^{+}_{2/4}(e^u)\,du \quad \text{has no ordinary convergence strip.} \end{gathered}} \tag{29}$$

The plus subtraction removes two different asymptotic tails of the complete source sum; neither remainder is an isolated index-zero Mellin transform. See Section 7.

Audit box, off the certified path: logic.

$$\boxed{\begin{gathered} \Re s > 1: \quad \int_{\mathbb{R}} e^{su/2} \frac{d}{du} H^{+}_{2/4}(e^u)\,du \\ = -\frac{s}{2} R^{+}_{2/4}(s) + e^{s^2/16} - \frac{2s}{s-1} e^{(s-1)^2/16}, \\ \int_{\mathbb{R}} e^{su/2} \frac{d}{du} \chi(u)\,du = e^{s^2/16}, \\ \int_{\mathbb{R}} e^{su/2} \frac{d}{du} [2e^{-u/2}\chi(-u)]\,du = -\frac{2s}{s-1} e^{(s-1)^2/16}. \end{gathered}} \tag{30}$$

Only the derivative integral has this initial plus half-plane; its continuation is supplied by the entire remainder. See Sections 7 and 8.

Audit box, off the certified path: logic.

$$\boxed{\begin{gathered} Q_{1/4}(s) := \frac{1-s}{2} e^{s^2/16} + \frac{s(s-1)}{4} R_{1/4}(s), \\ Q_{1/4} : \mathbb{C} \to \mathbb{C} \quad \text{entire}, \\ Q_{1/4}(s)\Big|_{s=0} = \tfrac{1}{2}. \end{gathered}} \tag{31}$$

The original zero mode survives as the tracked residue; it is not the claim that its isolated Mellin integral converges. See Section 8.

Audit box, off the certified path: logic.

$$\boxed{\begin{gathered} Q_{1/4}(s) = \frac{1-s}{2} \int_{-\infty}^{\infty} e^{su/2} \frac{d}{du} H_{1/4}(e^u)\, du, \\ \int_{-\infty}^{\infty} \frac{d}{du} H_{1/4}(e^u)\, du = 1 - 0 = 1. \end{gathered}} \tag{32}$$

The derivative is an equivalent completed readout, not a claim that the original n=0 term has an independent convergent Mellin integral. Its boundary jump records that retained constant. See Section 8.

Audit box, off the certified path: logic, standard.

$$\begin{gathered}\int_0^\infty x^{s/2}\frac{d}{dx}H_{1/4}(x)\,dx\\ =(1+2^{s-1}-2^{2s-1})\int_0^\infty x^{s/2}\frac{d}{dx}\Theta_0(x)\,dx,\\ \int_0^\infty x^{s/2}\frac{d}{dx}\Theta_0(x)\,dx=-s\pi^{-s/2}\Gamma(s/2)\zeta(s),\\ \Re s>1\quad\text{(auxiliary proof domain)},\\ A_{1/4}(s):=(1-2^{s-1})(1+2^s),\\ Q_{1/4}(s)=A_{1/4}(s)\xi(s),\\ A_{1/4}(s)\neq 0\quad\text{if } 0<\Re s<1.\end{gathered}\tag{33}$$

Derived identity, not printed verbatim in NIST. The auxiliary positive-index evaluation identifies the same entire readout; it does not redefine the bilateral source. See Section 8 and [4, 8, 6].

Audit box, off the certified path: logic, standard.

$$\begin{gathered}Q^-_{2/4}(s):=\frac{1-s}{2}e^{s^2/16}+\frac{s(s-1)}{4}R^-_{2/4}(s),\\ Q^+_{2/4}(s):=\frac{1-s}{2}e^{s^2/16}+se^{(s-1)^2/16}+\frac{s(s-1)}{4}R^+_{2/4}(s),\\ Q^+_{2/4}(s)=2^s\xi(s),\quad Q^-_{2/4}(s)=(2-2^s)\xi(s),\\ Q^+_{2/4}+Q^-_{2/4}=2\xi,\qquad Q_{1/4}(s)=\tfrac{1+2^s}{2}Q^-_{2/4}(s),\\ Q^+_{2/4}(0)=Q^-_{2/4}(0)=\tfrac{1}{2}.\end{gathered}\tag{34}$$

The plus extra Gaussian term is the dual-end tail. Both entire scalar readouts share classical xi; neither recovers indexed quarter orientation. See Section 8.

Audit box, off the certified path: logic, standard.

$$\begin{gathered}Q^-_{2/4}(-s)=\frac{1+s}{2}e^{s^2/16}+\frac{s(s+1)}{4}R^-_{2/4}(-s)=(2-2^{-s})\xi(1+s),\\ Q^+_{2/4}(-s)=\frac{1+s}{2}e^{s^2/16}-se^{(s+1)^2/16}+\frac{s(s+1)}{4}R^+_{2/4}(-s)=2^{-s}\xi(1+s),\\ 2^s\neq 0,\quad 2-2^s=0\Longrightarrow\Re s=1,\\ \Re s>0\Longrightarrow Q^+_{2/4}(-s),Q^-_{2/4}(-s)\neq 0.\end{gathered}\tag{35}$$

Both reflected zero-free statements use the classical Re(1+s)>1 domain. They do not prove the original RH zero-free condition; see Section 9.

Audit box, off the certified path: logic.

$$\boxed{\begin{array}{c} A_{1/4}(0) = 1, \qquad Q_{1/4}(0) = \tfrac{1}{2}, \\ 1 - 2^{s-1} = 0 \implies \Re s = 1, \\ 1 + 2^{s} = 0 \implies \Re s = 0. \end{array}} \tag{36}$$

Unlike the earlier paired factor, this multiplier has no artificial zero at s=0. Its added boundary zeros prevent an all-zeros-on-line claim for Q. See Section 9.

Audit box, off the certified path: logic.

$$\boxed{\begin{array}{c} s = \sigma_{\mathrm{sp}} + it, \quad \sigma_{\mathrm{sp}} = \Re s,\ t = \Im s, \quad m \in \mathbb{Z}, \\ 1 - 2^{s-1} = 0 \iff \sigma_{\mathrm{sp}} = 1, \quad t = \dfrac{2\pi m}{\log 2}, \\ 1 + 2^{s} = 0 \iff \sigma_{\mathrm{sp}} = 0, \quad t = \dfrac{(2m+1)\pi}{\log 2}, \\ 2 - 2^{s} = 0 \iff \sigma_{\mathrm{sp}} = 1, \quad t = \dfrac{2\pi m}{\log 2}. \end{array}} \tag{37}$$

The first and third added-zero loci coincide on Re(s)=1; the second lies on Re(s)=0. None lies inside the open critical strip. See Section 9.

Audit box, off the certified path: logic, standard.

$$\boxed{\begin{array}{c} Q^{+}_{2/4}(s) = 2^{s}\xi(s), \quad 2^{s} \neq 0 \quad (s \in \mathbb{C}), \\ \mathrm{Zeros}(Q^{+}_{2/4}) = \mathrm{Zeros}(\xi) \quad \text{with multiplicities}, \\ \widehat{Q}^{+}_{2/4}(z_{\mathrm{sp}}) := Q^{+}_{2/4}(\tfrac{1}{2} + iz_{\mathrm{sp}}), \quad \widehat{Q}^{+}_{2/4} \text{ is entire in } z_{\mathrm{sp}}, \\ \Re s = \tfrac{1}{2} - \Im z_{\mathrm{sp}}, \quad \Im s = \Re z_{\mathrm{sp}}, \\ \mathrm{RH} \iff \text{every zero of } \widehat{Q}^{+}_{2/4}(z_{\mathrm{sp}}) \text{ has } \Im z_{\mathrm{sp}} = 0. \end{array}} \tag{38}$$

The spectral z coordinate is separately registered from Jensen's Taylor z. This is an equivalent global zero condition, not a proof of RH or recovery of an indexed source map. The quarter and minus carriers have added boundary zeros. See Section 9.

Audit box, off the certified path: logic, standard.

$$\boxed{\begin{array}{c} \mathrm{RH} \iff Q_{1/4}(s) \neq 0 \quad \text{for all } \tfrac{1}{2} < \Re s < 1, \\ \mathrm{RH} \iff \text{all zeros of } \widehat{Q}^{+}_{2/4}(z_{\mathrm{sp}}) \text{ have } \Im z_{\mathrm{sp}} = 0, \\ \text{equivalence of conditions} \quad \neq \quad \text{proof of the zero-free condition.} \end{array}} \tag{39}$$

Terminal analytic statement of the scalar chain only; since the revision of 13 October 2026 it is off the certified path (Section 11). See Sections 9 and 10. H-AUD083 and H-VER154/156 already own their separate HG material; none is re-derived or promoted here. [23, 24, 25].

Provenance: logic.

$$\boxed{\begin{array}{c}(n,a)\mapsto 4n+4a:\ \mathbb{Z}\times\{0,1/4,2/4,3/4\}\to\mathbb{Z}\text{ bijective},\quad p_0(4n+4a,x)=p_a(n,16x),\\ (p_0,p_{2/4},p_{1/4},p_{3/4})\longleftrightarrow(\mathsf{A},\mathsf{B},\mathsf{E},\mathsf{O})\text{ invertible at each }(n,x),\\ \mathsf{O}(-n-1,x)=-\mathsf{O}(n,x),\qquad \mathsf{O}(0,x)\neq 0,\\ \text{certified: the indexed four-rail packet, term by term.}\end{array}} \tag{40}$$

The certified sink since the revision of 13 October 2026. An exact equivalence does not restore what a sum has cancelled; see Section 11 for the reason, in full, and (39) for the equivalence itself.

1 Reader contract and the index-zero objection

Revision of 13 October 2026. The paper is downgraded: its exact RH equivalence passes through a scalar readout that loses information, so it is no longer certified. The certified content is the indexed four-rail packet. The reason is set out in Section 11; nothing proved below is withdrawn.

Status of this section. This is the first section-by-section manuscript draft for review. The received Q&A is retained in its live source file and in Git history [21]. Section 0 remains the equation registry during staged expansion; its explanatory notes are not yet removed.

What the original objection actually asks. The opening analogy was between a quotient such as tangent, which may be undefined at zeros of its denominator, and its separate sine and cosine components. The decisive clarification came in the user's next objection: "it's not the riemannZeta(0) itself that's the problem"; the objection concerns sums and products whose *index set* skips zero [21]. The claim is not that the classical Riemann hypothesis (RH) is ill-formed. It is a request for a target in which the discrete zero mode is defined before any transform and its subsequent fate can be checked.

The de Vries wave motivation, its subsequent bridge-centered RG closure, and the Jordan order-3 half-shift source are documented in distinct author manuscripts [14, 15, 16]. Those sources establish the provenance of the DVJ question; they are not cited as independent proofs of the new quarter-characteristic RH equivalence derived in this paper.

Three different origins. Equation (1) includes the original integer index $n = 0$ with Gaussian value one. The Fourier-dual index $k = 0$ is a different coordinate: its coefficient may vanish as the result of the quarter-characteristic cancellation in (19). Finally $u = 0$ is the logarithmic coordinate at $x = 1$, not the discrete lattice index zero; see (13). None of these may be replaced by a convenient value of $\xi(0)$, a Taylor constant or a post-hoc completion term. Their distinct roles govern the paper.

What counts as retaining a mode. The full theta series must first be an honest bilateral sum over $\mathbb{Z}$, as in (1). A positive-index series may represent the same completed analytic function while failing this separate source-bookkeeping requirement. In particular, DLMF 25.5.13–14 contains an explicit subtraction of the original constant mode, as (4) records [4, 5]. The isolated constant has no ordinary Mellin integral over the entire positive half-line ((5)); a new representation must account for this fact, not silently divide by its index or assign its answer after the subtraction.

Keep the mistakes, but do not use them as premises. The Li-series inference in (6) is a valid classical criterion followed by an invalid inference about the Gaussian mode. The artificial central coefficient in (8) was inserted after subtraction. The reflection-matched multiplier in (9) introduced a new endpoint zero. All three are preserved as labelled historical FIREWALL nodes. They explain why the subsequent construction was needed, but they have no dependency edge into any box outside the firewall: neither into the certified sink nor into the audit chain that ends in the RH equivalence (revision of 13 October 2026, BE 72f/dfd3). Failure of a proposed *source identification* does not invalidate its separately established classical identity.

The coefficient-index caution: Pólya–Jensen. The sequence γ_ν is *defined*, not assumed, by the even Taylor expansion of completed xi about its symmetry point in (7). The literature usually calls its index n; here ν marks it as a Taylor index distinct from the original lattice index n. In particular the Taylor coefficient at index zero is $8\xi(1/2)$, not the Gaussian zero-mode value one. The associated finite Jensen polynomials test hyperbolicity at every degree and shift. That is a genuine RH equivalent, but it does not certify preservation of the original Gaussian lattice mode [11].

Positive output and the stopping boundary. The sums over n of the recursive parity split (10) and (11), collected in (12), yield the kernel in (14). Its original constant tail and its dual-index-zero cancellation are distinguished in (23). Completion occurs at the level of the whole logarithmic integral in (27), giving the entire function in (31). The exact factor identity (33) leads to the RH-equivalent zero condition (39). This chain proves an equivalent *formulation*; it does not by itself prove the required zero-free condition.

Source boundary. H-AUD083 and H-VER154/156 retain their previously established hypergroup source ownership. H-PAP003 records the analytic bridge and its failure modes without creating a new HG dependency or silently reproving the pre-existing De Vries–Jordan transport [23, 24, 25]. The exact information-loss accounting and independent verification belong to the later sections and appendices. No new source or theorem status is promoted by this staged prose draft.

2 Classical equivalents and first wrong turn

Status of this section. This section records the original search and its first mistaken answer. The classical criteria are not rejected as theorems. What is rejected is the inference that a regular completed value, a local Taylor coefficient, or a zero-set equivalence by itself preserves the original Gaussian lattice index zero. The sequence follows the received Q&A [21]; the source identities are checked against their named classical owners. The historical formulation is Riemann's 1859 one [2].

The first answer: move to an entire function. The original question asked for a Gaussian half-shift, quarter-turn, or logarithmic coordinate avoiding division by zero. The answer began with completed Riemann xi, (3), whose functional equation and entireness are standard [6]. This is a valid replacement when the particular defect is a reciprocal or logarithm evaluated at a zero of zeta. In the rotated spectral coordinate, the condition that all zeros of completed xi lie on the symmetry line becomes a real-zero statement for an entire function. It is a genuine RH equivalence, but it has not yet said which *discrete Gaussian summand* supplied the completed function.

The initial answer then observed that $\xi(0) = 1/2$, or $2\xi(0) = 1$, and treated this finite completed value as an answer to the proposed De Vries–Jordan (DVJ) α^0 coordinate. That was an invalid source-identification inference. The argument $s = 0$ in $\xi(s)$ is not the integer summation index $n = 0$ of (1). A completed function can be entire while its displayed theta input has already subtracted the central Gaussian. The explicit instance is (4); its provenance is reviewed in Section 3.

A quarter-turn is not a quarter characteristic. The classical rotation of the spectral argument to a centered variable is useful for the all-real-zeros statement. It is not the operation $n \mapsto n + 1/4$ on the Gaussian lattice. Likewise, the de Bruijn–Newman Gaussian heat-flow equivalent is a valid zero-location criterion, including the established non-negativity of its deformation constant [13]. Its standard theta-derived Fourier kernel, as recorded in the circulated Q&A, is written with positive Gaussian indices. Neither spectral rotation nor heat deformation supplies, without an additional source map, a certificate for the original lattice mode. We therefore preserve these as classical comparison routes rather than identify them with the new quarter-characteristic carrier.

Li's legitimate local logarithm. Li's theorem gives a criterion in which a sequence of logarithmic coefficients is nonnegative at every positive integer index [12]. The Q&A used the functional equation to center a locally analytic logarithm at the regular spectral point $s = 0$. Equation (6) records the resulting series and the exact point where its use in our discussion became invalid. The branch is local around a nonzero completed value; there is no need to evaluate $\log 0$ there. That fact establishes local analyticity, *not* preservation of a Gaussian index-zero summand.

The answer initially described $\mathcal{L}(0) = 0$ as a candidate α^0 witness. The user immediately objected that the series has the coefficients λ_n/n indexed from $n = 1$ upwards, exactly the representation one was trying not to substitute for a bilateral zero-inclusive Gaussian. The failure is not Li's criterion; it is the attempted identification of its normalized constant Taylor value with the original $n = 0$ Gaussian. Accordingly (6) is a FIREWALL node, not an incoming proof edge to (39).

Pólya–Jensen: an index zero of a different kind. The other classical suggestion was the all-degree, all-shift hyperbolicity criterion for the Jensen polynomials of completed xi [11]. Its coefficient input is now explicitly defined in (7). We use the Taylor index ν, not the lattice index n, to prevent accidental identification. The Taylor expansion does start at $\nu = 0$, and its constant coefficient is $8\xi(1/2)$. That does not make it the unshifted Gaussian summand of value one. All degrees and shifts are required by this classical criterion: selecting only the first two or three Jordan levels cannot silently replace its universal quantifiers.

The Riesz and Báez-Duarte alternatives discussed in the Q&A provide other exact RH-equivalent coefficient or growth tests. Their safe nonzero zeta arguments do not settle the separate index-provenance question. We cite them here only as recorded historical alternatives, not as load-bearing identities of this paper [21].

The half-shift multiplier: correct zeros, wrong origin. A different suggestion began from the Jacobi even/odd split in (10). The half-shifted Hurwitz identity gives a factor involving $2^s - 1$ [9]. Multiplying by its reflected partner leads to (9). This factorization is an exact scalar zero-set comparison within the open critical strip: its factors do not vanish there. However, the multiplier introduces an endpoint zero at $s = 0$, although completed xi is nonzero there. The user did not ask merely for a function with an RH-equivalent open-strip zero condition; the original coordinate also had to be traceable. Consequently this paired scalar is a valid comparison and a failed proposed DVJ normalization, both at once.

What the objection forced us to distinguish. The correction was explicit: "it's not the riemannZeta(0) *itself* that's the problem" [21]. The important index is $n \in \mathbb{Z}$, with a defined original term at $n = 0$, not a spectral argument, a Taylor index, the Fourier-dual index, or the origin of a logarithmic coordinate. An entire function or an RH equivalence alone cannot be used as a backward reconstruction theorem for that source term. Nor may an independently assigned central coefficient stand in for a mode whose transform has not been justified.

The first question was therefore answered too weakly. We must keep the entire classical equivalences in their proper role, preserve the failed inference on the record, and investigate the theta source before declaring a zero-mode result. Section 3 takes up the actual NIST formula and the next mistake: assigning a central coefficient after the original central Gaussian was already subtracted. None of the classical routes in this section becomes a new HG theorem, a new dependency edge, or a proof of RH.

3 The NIST subtraction and artificial central coefficient

Status of this section. Documentary reconstruction of the second wrong turn, with the standard formulas separated from what the subsequent interpretation claimed. The imported Q&A is the provenance of the conversation, not an independent authority for the classical formula [21, 4, 5].

What NIST actually says. The question was immediate: "does it actually go from 0 to inf?" The completed integral in DLMF 25.5.13 starts at the *continuous* point $x = 1$, not zero; equation (4) records its exact limits. DLMF 25.5.14 uses $\omega(x)$, a sum over positive integer indices, and equivalently half of the complete theta lattice *minus one* [4, 5]. The subtraction is precisely the original $n = 0$ term of (1). It is a mathematically valid standard representation of completed xi, but not the source-mode retention requested in this paper.

Two distinctions were conflated in the earlier answer. The continuous Mellin coordinate may range over $(0, \infty)$ while its discrete Gaussian sum still omits zero. Conversely, an expression may contain a bilateral $\mathbb{Z}$-sum while deleting its central summand algebraically. Neither property by itself proves that the original central mode survives a transform. Here the exact step is the passage from Θ_0 to $\Theta_0 - 1$, not the behavior of $\zeta(0)$.

The tempting reconstruction. The first attempted repair put a Kronecker central coefficient into the completed NIST expression, as preserved in (8). With the central indicator inserted, the term indexed by $n = 0$ becomes $1/2$. For every nonzero index, the Gaussian integral over $[1, \infty)$ is well defined, and summing the resulting entries reconstructs the same completed xi. Thus the attempted K_n is not an incorrect algebraic repackaging. The incorrect claim was that the assigned $1/2$ constitutes the *Mellin image of the unremoved central Gaussian*. The Gaussian was already subtracted inside the integrand; the completion constant was subsequently allocated to its vacated index [21].

Why the isolated source mode cannot be transformed termwise. The original central Gaussian equals one for every positive x. Its unregularized Mellin integral has incompatible convergence requirements at the two ends, exactly as stated in (5). This is a domain obstruction for an isolated constant Mellin transform. It is neither an impossibility theorem for the Riemann hypothesis nor a license to replace the central summand with an unrelated completion constant. A proof must specify a whole-kernel transformation and the treatment of its asymptotic contribution before claiming to have transported the original index-zero information.

Retraction and the resulting design constraint. The circulated discussion explicitly retracted the central-witness interpretation of K_n [21]. The equation remains in Section 0 as a labelled FIREWALL: it teaches exactly why a completed analytic value must not be confused with a source coefficient. This error also explains why simply presenting Li's regular constant, the Pólya–Jensen Taylor coefficient, or the reflection-matched scalar from Section 2 would not settle the user's question.

The next move is therefore made *before* the Mellin operation: refine the full bilateral theta lattice by repeated parity splitting, retain the original unshifted branch, and build a combination whose *dual* constant coefficient cancels. The exact split is in (10) and (11). Section 4 develops it without

a post-hoc value assignment. No new HG proof owner, BE status, or repository theorem DAG edge is introduced here.

4 Two half-shifts and four quarter cosets

Status of this section. Exact elementary reindexing of the bilateral Gaussian source. The user proposed a "quarter-step (in logarithmic coordinates)" after identifying the missing index-zero term in the NIST presentation [21]. The operative change is to split the *source lattice before* applying the Mellin transform, not to restore an index after a positive-index readout has already discarded it. The earlier DVJ half-shift and D12 Gaussian source are recorded in the DVJ Jordan manuscript and Paper 5 [16, 17]; the four-coset reindexing below is proved here and is not attributed to those earlier works.

First split: even and odd integers. Begin with the full Θ_a in (1). Partition its original index into $n = 2m$ and $n = 2m+1$, with the new index m running through *all* integers in both branches. The even contribution has characteristic $a/2$ at scale $4x$; the odd contribution has characteristic $(a+1)/2$ at the same scale. This proves the general identity in (10), including its zero-characteristic specialization [7]. No endpoint is deleted. The original summand $n = 0$ belongs to the even branch as $m = 0$, where its value is still one.

Second split: both descendants. Apply that same identity to the even branch and independently to the odd branch. The even branch contributes the zero and half characteristics at scale $16x$; the odd branch contributes the quarter and three-quarter characteristics at that scale. Collecting all four yields (11). This is an identity of complete bilateral series: each coset is indexed by all $m \in \mathbb{Z}$, and their underlying integer classes are disjoint and exhaustive. It is not a claim that four independent new spectra have appeared.

The same split, term by term. Both splits are reindexings before they are sums. Since $(4m+4a)^2 x = (m+a)^2\, 16x$, each term satisfies $p_0(4m+4a, x) = p_a(m, 16x)$ for $a \in \{0, 1/4, 2/4, 3/4\}$, and $(m, a) \mapsto 4m + 4a$ is a bijection from $\mathbb{Z} \times \{0, 1/4, 2/4, 3/4\}$ onto $\mathbb{Z}$. The first split is the case $a \in \{0, 2/4\}$ at scale $4x$. So every term of the original index goes to exactly one term of one coset, and back. This per-index form is what Section 0 certifies, (10) and (11); the series identities above are its sums, (12), and are kept off the certified path, because a sum over the index cannot see the odd quarter channel (Section 11).

At the source index zero, the unshifted branch still contains the constant Gaussian one. By contrast, the *reindexed* zero terms of the two quarter-characteristic branches have the finite, nonconstant values displayed in (11). Those terms correspond to source classes of integers congruent to one and three modulo four; neither is the original unshifted zero mode. This distinction is why our notation keeps the original index n and the characteristic a separate.

The four indexed rails, before scalarization. The actual source vector in (15) contains p_0, $p_{2/4}$, $p_{1/4}$, and $p_{3/4}$ at each fixed n and x. Its block-diagonal four-by-four change of coordinates gives A, B, E, and O. Each two-by-two block has determinant minus one-half after the factor of one-half is included, so its displayed inverse reconstructs all four original indexed values. The former three-by-three inverse only reconstructed the chosen three inputs and did not retain the half-rail or quarter orientation as a four-rail packet. This is an identity of finite Gaussian coordinates, not a claim of a new hypergroup (HG) source-to-theta intertwiner.

The scalar $K^{\mathrm{idx}}_{1/4}$ selects the unshifted rail minus E. Summing it yields $H_{1/4}$, but that scalarization loses O and cannot invert the four-coordinate packet. As a check, the indexed $\mathsf{O}(0,x)$ is nonzero at positive scale, although its complete bilateral sum is zero by reflection. The full matrix and inverse, not the scalar readout, are the source-level ownership statement in (15).

Equal theta values do not erase labelled cosets. The full quarter and three-quarter theta functions are equal under the bijection $m \mapsto -m-1$. The equality of their *sums* does not identify their terms with the same displayed index, nor does it authorize removing either labelled residue class from the four-way split. The two labelled sectors are kept as provenance until their coefficients are explicitly combined. The latter operation will be needed for the Fourier character in Section 5.

The scale-change identity. The union of the quarter and three-quarter branches at scale x consists exactly of the odd integer terms of the unshifted theta series at scale $x/16$. Subtracting its even-index subseries gives the first identity in (17); substituting it into (14) gives the second. This is an exact algebraic cross-check for the eventual completed Xi factor, not permission to Mellin-transform the three full theta terms separately. Each has a constant original tail, and their isolated Mellin integrals do not possess a common ordinary convergence strip.

The half-rail pair is a separate two-coordinate cut. Equation (22) takes the sum and difference of the p_0 and $p_{2/4}$ rails. Before summing over n, these two operations are precisely twice A and twice B. Their inverse reconstructs the same indexed zero/half pair; it does not touch the two quarter rails. The whole-sum identities then follow from the first parity split: $H^{+}_{2/4}(x)$ equals the full theta at quarter scale, while $H^{-}_{2/4}(x)$ is twice the unshifted theta minus that quarter-scale theta. They are two *scalar readouts* of an invertible subpacket, not a three-or-four-coordinate source inverse.

At displayed source index zero, the plus and minus summands are, respectively, one plus and one minus the half-rail Gaussian $e^{-\pi x/4}$. Both *complete* kernels tend to one as $x \to \infty$; the small-scale dual constants differ and are evaluated only after full Poisson summation in Section 5. This explicit indexed asymmetry is retained when the two entire completed functions are constructed later. Neither function reconstructs O.

The even quarter sum is not one half. Equation (21) distinguishes the scalar quarter-even sum from its prefactor. The reflection $n \mapsto -n-1$ swaps the quarter and three-quarter indexed Gaussian rails. Their two complete sums are equal, but the indexed even and odd components are independent before summation. Splitting the odd integers into the two quarter cosets proves that the even sum is one half of the complete half-characteristic theta at scale $x/4$. The second parity identity expresses it as a difference of unshifted bilateral thetas at scales $x/16$ and $x/4$. The coefficient $1/2$ is not a constant value of the sum. $\sum_{n\in\mathbb{Z}} \mathsf{O} = 0$, but the nonzero indexed odd coordinate must remain in the invertible four-rail source packet. This is a scalar information-loss readout, not a source-preserving HG/DVJ map [22].

A distinct quarter in logarithmic coordinates. Set $x = e^u$, with $u \in \mathbb{R}$. Then $dx/x = du$, so the whole positive scale line becomes a whole real logarithmic line. Equation (13) records the paired

Mellin power at the centered spectral argument: its quarter exponent follows from the functional-equation midpoint, not from the quarter coset labels. The logarithmic origin $u = 0$ corresponds to $x = 1$, not to $\log 0$. Thus the quarter characteristic, the quarter logarithmic weight, and the original integer-index zero are separately typed objects.

What this split has and has not achieved. All original integers, including zero, survive the two parity splits. The operation is reversible by recombining all four cosets; it creates no division by the index. It does not, however, make the Mellin transform of an *isolated* constant converge. That obstruction remains (5). The next constructive choice is the quarter-characteristic difference in (14), designed so that its Fourier-*dual* constant cancels while its original unshifted constant remains.

5 Original and dual zero-mode accounting

Status of this section. Exact Gaussian and Fourier bookkeeping for the quarter-characteristic difference. The transformation acts on complete theta series; the original and dual zero indices are not identified. The subsequent Mellin presentation remains the subject of Section 6.

The difference is constructed before transformation. The selected kernel $H_{1/4}$ is (14): the unshifted full theta minus the mean of the quarter- and three-quarter-characteristic full thetas. It is not DLMF's ω; that function subtracts the original central term explicitly. Here the unshifted summand at $n = 0$ remains exactly one, and the two shifted families are retained with their own indices. This distinction responds to the user's original objection [21, 5].

The logarithmic per-index package in (18) exhibits the actual contribution $G_{1/4}(0, u)$. It contains the original constant one *and* the nonconstant quarter-characteristic terms at the same displayed integer index. It is not itself equal to the full kernel, which requires the complete bilateral sum. In particular, taking a limit of this one summand at the small-scale end is not a substitute for taking the limit of the summed kernel; the cancellations there involve the whole lattice.

Four distinct Fourier phases and indexed reflection. Equation (20) gives the complete zero, quarter, half, and three-quarter phases as one, i^k, $(-1)^k$, and $(-i)^k$. Their symmetric combinations give an even/odd parity filter for the first two source rails and cosine/sine Fourier channels for the last two. Oddness cancels the complete O sum under paired Fourier indices, but the *indexed* odd rail is not zero; the per-index statements are (16). In particular $p_{2/4}(0, x)$ is $e^{-\pi x/4}$, whereas the original $p_0(0, x)$ equals one. Quarter and three-quarter reindexed zero terms are different again. The index reflection $n \mapsto -n - 1$ exchanges the quarter labels and changes the sign of the indexed odd quarter channel, without annihilating that source coordinate.

Shifted Poisson summation, with all dual indices. For positive x, the Gaussian is a Schwartz function, so ordinary shifted Poisson summation applies to each full theta. The shift a supplies the Fourier coefficient $e^{2\pi i k a}$. At zero characteristic it is one; for the two quarter shifts their average is the real character $\cos(\pi k/2)$. Combining these exact coefficients produces (19), whose sum is explicitly over $k \in \mathbb{Z}$ [8, 7, 10].

The cancellation at $k = 0$ is not an omitted summation index: $W_{1/4}(0) = 0$ follows from the coefficient of a Fourier mode which was present in all three transformed theta series. Other multiples of four also carry zero weight; indices congruent to one or three have weight one, and those congruent to two have weight two. The full dual series is retained even at indices with zero coefficient. It is therefore important to say "dual-mode cancellation", not "original-mode deletion". Poisson summation mixes the source coefficients; it does not identify the original $n = 0$ summand with the dual $k = 0$ coefficient.

Quarter-even Fourier zero coefficient and odd loss. The full quarter-even identity (21) has Fourier weight $\cos(\pi k/2)$. In particular the *dual* $k = 0$ weight is 1, contributing $x^{-1/2}$, not a scalar $1/2$. Nonzero dual indices supply the remaining theta terms. The odd weight is $i \sin(\pi k/2)$; its complete bilateral sum vanishes by the pairing $k \mapsto -k$. This is the Fourier image of the original indexed reflection; it does not make $\mathsf{O}(0, x)$ zero. Neither this summed E nor the identically zero

summed O reconstructs the four indexed rails. The source map remains BE `72f/8ae`, separately OPEN [22].

The separate half-rail dual-zero ledger. Applying shifted Poisson summation to the two *complete* half-rail sums in (22) gives the weights one plus or minus $(-1)^k$. The dual $k = 0$ coefficients are two in the plus channel and zero in the minus channel. Both sums still run over all integers. The plus channel therefore behaves as twice the inverse square root of scale at $x \to 0^+$, while the minus channel is exponentially small there. At the original large-scale end, each complete kernel tends to one. It would be incorrect to apply the one-tail compensator of the minus channel to the plus channel: the latter has a second, independent dual-end asymptotic contribution. This difference is carried into (29) in Section 7.

The original zero terms and the dual zero coefficients are not two descriptions of the same indexed source term. In particular, Poisson summation sends an entire Gaussian lattice to the dual coefficient sequence; one may not infer source retention from a single scalar Fourier weight. Even the two half-rail readouts leave the indexed O coordinate untouched. The full four-rail source packet remains (15).

Asymptotics obtained on their correct sides. At large positive scale, the unshifted theta tends to one; the leading quarter-shift Gaussian has exponent $-\pi x/16$. Therefore the full kernel tends to one with the exponential error recorded in (23). This is the retained original constant tail, not an assigned completion coefficient.

At small positive scale, use the *dual* representation instead. Its zero coefficient vanishes and its first nonzero indices are $k = 1$ and $k = -1$, each of weight one. The remaining dual terms are exponentially smaller. The bound in (23) follows directly, and has no constant or inverse-square-root-only term. Both asymptotic statements concern the complete kernel, not separate Mellin transforms of its individual Gaussian summands.

Convergence and ownership of the Mellin domain. The two tails give the exact initial convergence half-plane $\mathcal{D}_{1/4}$ in (23) for the whole-kernel logarithmic Mellin integral (24). At the small-scale end the exponential suppression absorbs any fixed complex power; at the large-scale end the retained constant tail requires $\Re s < 0$. At $s = 0$ the raw integral has no ordinary convergence, as Section 0 already warns. This fact is consistent with retaining $n = 0$; it is not silently turned into a convergent isolated zero-mode integral.

What is and is not licensed downstream. The original index-zero Gaussian has been tracked through the full source kernel, while the distinct dual zero coefficient vanishes by a stated Fourier identity. The result is an exact, non-termwise basis for continuation. It does *not* license dropping the full bilateral sum in favor of the positive-index Dirichlet expansion, even if that expansion subsequently verifies the same meromorphic function. The historical relapse is retained in (25) and developed next in Section 6. Nor does this section claim that the entire-carrier equivalence proves RH; that separate boundary is (39).

6 The termwise Mellin and positive-index regression

Status of this section. The positive-index formula is an exact derived cross-check in its stated domain. Its attempted use as *the* source representation was retracted. This section preserves that mistake and the user's objection, rather than silently rewriting the development as a straight-line derivation [21].

The starting point was already bilateral. Section 5 had established the complete source kernel (18) and its full dual lattice (19). The logarithmic Mellin transform (24) acts on the *summed* kernel, without deleting its $n = 0$ index or treating individual Gaussian terms as independent Mellin integrals. The exact ordinary convergence domain is $\mathcal{D}_{1/4}$, fixed by (23). In particular, the zero-coordinate value of the raw transform is not obtained by substitution at $s = 0$.

Where the presentation regressed. The next answer used the dual Gaussian and exchanged sum and integral, obtaining (25). For each nonzero dual frequency, substitution of the reciprocal Gaussian variable evaluates a Gamma integral. Pairing positive and negative Fourier frequencies then writes the result with k beginning at one. The interchange and resulting Dirichlet series are justified in $\Re s < 0$: the Gaussian-integral weights have the absolute summability required there. Thus the equation is a legitimate cross-check *of the same meromorphic object*; it is not an unjustified numerical approximation.

The user's immediate correction was explicit:

> "Using the dual Gaussian expansion, one obtains" – and how do we express m_1/4 not in terms of sum(k=1..inf)? it's so ridiculous!

The wording is preserved from the received Q&A [21]. The crucial point is not that a convergent positive-index Dirichlet series cannot exist. It is that the proof programme had deliberately chosen a source whose original index zero was *present*, and the assistant presented a readout in which zero was again absent.

Two different zero-index statements. In the original lattice, (18) explicitly retains the constant Gaussian at $n = 0$, alongside two shifted Gaussians at that displayed index. In the dual lattice, the coefficient $W_{1/4}(0)$ is exactly zero by Fourier cancellation (19). Passing from the $k \in \mathbb{Z}$ dual series to its paired positive-frequency readout suppresses the *display* of the second statement. It does not retroactively prove that the original $n = 0$ mode was absent, nor that its isolated Mellin integral exists. Those would be false inferences.

There is no contradiction between (i) equality of the readout functions on a shared domain and (ii) loss of the original per-index provenance from the readout expression. The FIREWALL classification of (25) records exactly that distinction. The readout can identify a completed function later; it cannot be substituted for the source kernel when testing the DVJ zero-mode transport requirement.

The correction accepted for the subsequent construction. Keep the integral over the whole logarithmic line and retain $\sum_{n\in\mathbb{Z}} G_{1/4}(n, u)$ inside its integrand, as in (24). Use the complete

dual series only for Fourier identities and convergence bounds. Do not commute the sum and Mellin integral when doing so forces an undefined individual $n = 0$ transform. A positive-index computation may reappear only as an *auxiliary identification*, and must say so.

This rule does not magically make the raw Mellin integral convergent at $s = 0$. The retained constant tail produces an authentic pole. Section 7 treats that tail at the level of the *whole* bilateral kernel, rather than assigning an artificial coefficient to the origin as in (8).

7 Bilateral logarithmic Gaussian completion

Status of this section. This section constructs an entire continuation of the *whole-kernel* Mellin expression, tracks the pole generated by the original constant tail, and gives a cutoff-independent derivative representation. The raw Mellin integral is not claimed to converge at its zero coordinate.

Start with the actual convergent object. Retain the equality in (24), including $n \in \mathbb{Z}$, as the definition of the raw transform in $\mathcal{D}_{1/4}$. The small-scale decay in (23) is faster than any fixed power, while at large scale the kernel tends to one. In logarithmic coordinates, this means that the full integrand has a constant tail as $u \to +\infty$, not an individually integrable central Gaussian. The zero coordinate of the raw integral is therefore a pole to be *accounted for*, not a value to be guessed.

Smooth compensation of the entire tail. Use the fixed Gaussian step χ in (26). It tends to zero at the negative logarithmic end and one at the positive end. Define $R_{1/4}$ by (27): subtract χ from the *complete sum*, not from one selected lattice summand. The original zero-mode contribution is still displayed in (18), and the subtraction now has a stated asymptotic owner.

For positive u, the difference between the kernel and one decays like a Gaussian in e^u; the complementary error-function tail is bounded by a constant times $e^{-u^2}/(1+u)$. At negative u, the Poisson-dual estimate of (23) is even smaller than an ordinary Gaussian in u, and the Gaussian step has the same error-function bound. These two estimates give the whole-line bound recorded in (27). After multiplication by $e^{su/2}$, every fixed complex s is dominated on compact parameter sets by an integrable Gaussian tail. The same holds after any number of parameter derivatives, which only insert powers of u. Consequently $R_{1/4}$ is entire, by locally uniform dominated convergence. This is a statement about the summed remainder, not termwise Mellin convergence.

Evaluate the compensator only in its genuine domain. The derivative of the Gaussian step is the normalized ordinary Gaussian. For $\Re s < 0$, integrate its moment by parts. The boundary term vanishes at both ends in this domain; the remaining ordinary Gaussian moment gives exactly (28). In particular, $\mathcal{M}_{1/4}$ agrees there with $R_{1/4}$ minus the explicit compensator moment. The identity supplies a meromorphic continuation of $\mathcal{M}_{1/4}$, whose residue at the origin is -2. This residue is fixed by the large-scale value one of the original full kernel. No value of 0^{-s} or $\log 0$ was needed.

The finite completed zero coordinate. Multiply the raw transform on its convergence domain by the completion factor already registered in (31). Substituting the compensator identity cancels the displayed pole, and defines $Q_{1/4}$ everywhere by the right-hand side of (31). Both summands there are entire. Direct substitution yields $Q_{1/4}(0) = \frac{1}{2}$, and *this* finite coordinate is inherited from the proven constant-tail residue, not injected as the arbitrary K_0 of (8). This distinction is exactly why the earlier apparently harmless insertion was rejected.

The two half rails require different subtractions. The independent plus and minus whole-kernel identities are (22). The minus dual zero coefficient vanishes, so the full $H^-_{2/4}(e^u)$ decays super-exponentially at the negative logarithmic end. It tends to one at the positive end. Consequently

its remainder in (29) subtracts only the original-end step $\chi(u)$. This is the same *pattern* as for $H_{1/4}$, but the minus remainder $R^-_{2/4}$ is a separately defined function; it is not the quarter-kernel remainder $R_{1/4}$.

The plus dual zero coefficient is two, giving the extra tail twice the inverse square root of scale. In logarithmic variables this is $2e^{-u/2}$ at the negative end, whereas the original positive-end constant remains one. Its raw transform would require $\Re s > 1$ on the negative end and $\Re s < 0$ on the positive end: there is no common ordinary convergence strip. Thus it is *not* legitimate to define the plus transform by borrowing the minus raw Mellin integral or its one-tail remainder.

The entire two-tail compensator $C^+_{2/4}$ in (29) consists of the positive-end Gaussian step plus the reflected step carrying the independently determined dual amplitude two. Subtracting it from the *whole* plus theta sum leaves a remainder which decays faster than every exponential at both ends. The same conclusion holds for the distinct minus remainder. Dominated convergence on compact spectral sets, including arbitrary parameter derivatives, proves the two entire remainders; no isolated $n = 0$ Mellin transform is used.

Plus derivative bridge and its actual initial domain. The plus derivative transform in (30) is ordinary only in $\Re s > 1$: differentiation removes the positive-end constant but retains the differentiated inverse-square-root dual tail. Differentiate the *whole* plus kernel, replace it by the derivative of its entire remainder and of the two explicit compensator terms, and integrate the former by parts. The remainder contributes minus one-half the spectral parameter times $R^+_{2/4}$; its boundary terms vanish in the auxiliary domain and then continue holomorphically.

The moment of the derivative of χ is the complete Gaussian moment already used in (28). The reflected dual-step derivative gives the last term of (30), including its *minus* sign and its factor $2s/(s-1)$. The auxiliary expression has a pole at $s = 1$: it is not an entire representation of the uncompleted derivative. The completed expression in (34) multiplies by the required factor, removes that pole, and retains the dual-tail Gaussian term with its correct positive coefficient. For the minus channel the derivative of the full kernel already decays at both logarithmic ends, so its derivative integral is entire. Both normalizations are independently checked at the spectral origin and are not a declaration that the two remainders agree.

A second, cutoff-independent representation. There is an additional audit of the completed function: (32) writes it as the Fourier– Mellin transform of the derivative of the *whole* logarithmic kernel. That derivative decays rapidly at both ends, so its integral is entire in s. In $\mathcal{D}_{1/4}$, integration by parts identifies it with $-s\mathcal{M}_{1/4}/2$, giving the same completed readout by analytic continuation. The integral of the derivative at zero equals the boundary jump $1 - 0 = 1$: the retained constant is thus independently accountable without any arbitrary smooth cutoff.

Differentiation annihilates an isolated constant as a function of u. Accordingly, the derivative formula is an equivalent *completed readout*, not a substitute for the original indexed source (18). Its boundary jump and the explicit compensator residue are the two independent audits of that original constant. Section 8 now identifies the entire readout exactly, without making a positive-index series its canonical definition.

8 Entire carrier and exact factor identity

Status of this section. An exact identity of entire functions is established. Its proof keeps the bilateral source (18) and the entire completion (31) as canonical objects. A positive-index Dirichlet evaluation enters only in a separate convergence domain to identify the derivative transform with the classical completed xi function. It does not become the source definition.

The exact quarter-coset scale identity. The two successive half-shifts in (11) give the quarter-characteristic identity (17). It expresses the *whole* source kernel as three full, unshifted theta functions at related scales. This is an exact equality at every positive scale; it does not authorize three separate ordinary Mellin transforms, because those individual transforms have no shared strip of convergence. The distinction is the technical correction to the historical termwise regression of Section 6.

An entire derivative transform makes the bridge. Use (32) to define the same $Q_{1/4}$ through the derivative of the *summed* kernel. The derivative is rapidly decreasing at both logarithmic ends, so that transform is entire. Differentiating the full-theta scale identity and changing the positive integration variable in each *derivative* integral yields the first equality in (33). This change of variable is legitimate for $\Re s > 1$, where the individual unshifted theta derivative integrals converge. Their scale factors are one, 2^{s-1}, and -2^{2s-1}, respectively.

This step must not be confused with the invalid direct splitting of the *raw* Mellin transform. The derivative removes the constant tail before any individual integral is considered; its boundary jump is separately fixed in (32). No claim is made that an isolated original $n = 0$ Gaussian has a convergent unregularized Mellin integral.

A separate globally zero-equivalent entire carrier. The half-rail plus completion is the entire product $Q^{+}_{2/4}(s) = 2^s\xi(s)$, registered in (34). Since the exponential multiplier never vanishes at any complex argument, the plus carrier and classical xi have the same zeros with the same multiplicities *everywhere*, not merely in the open critical strip. The rotated function and its coordinate map are registered in (38). The entire scalar identity does not establish an independent source intertwiner, RH zero-freeness, or evenness of the rotated plus function. This is the distinction between an exact equivalent condition and proof of that condition [22].

Independent classical identification. For $\Re s > 1$, the remaining unshifted theta derivative may be evaluated using the standard convergent Gaussian/Gamma calculation. This gives the second equality of (33) [4, 6, 8, 3]. In this *auxiliary calculation*, differentiating the constant Gaussian makes its derivative zero; the convergent positive-index Dirichlet evaluation then identifies the standard zeta factor. This is disclosed rather than disguised: it is an analytic identity check, not a second canonical representation for the bilateral source. The source-to-readout information boundary remains the one spelled out in Section 6.

Combining that derivative evaluation with the prefactor in (32) yields the completed ξ factor and the exact multiplier $A_{1/4}$ stated in (33). The elementary factorization there follows by expansion: the scale polynomial is $1 + 2^{s-1} - 2^{2s-1}$, exactly $(1 - 2^{s-1})(1 + 2^s)$. These are *equalities*, not a numerical fit or an assumption of the Riemann Hypothesis. Since $Q_{1/4}$ and completed xi are both

entire, agreement on the open auxiliary half-plane extends their identity to every complex s by the identity theorem.

Exact completed half-rail pair. Apply the derivative readout of (32) separately to the two whole theta kernels in (22). The plus kernel is the unshifted theta at quarter scale; changing the variable in its derivative integral gives the factor 2^s. The minus kernel is twice the unshifted theta minus that same quarter-scale theta and gives $2-2^s$. In the common auxiliary domain $\Re s > 1$, both are evaluated by the established unshifted derivative–xi identity. The minus derivative is already entire; the plus expression is analytically continued through its separate two-tail remainder. Hence the two identities of entire functions in (34) hold globally by the identity theorem, not by termwise Mellin integration of the original indexed Gaussians.

For the minus formula the positive-end Gaussian step supplies the one-tail term. For the plus formula the negative-end inverse-square-root tail supplies the *additional* term with s times the reflected Gaussian moment. These are exactly the distinct completions derived in (29) and (30). No step sets $R^+_{2/4}$ equal to $R^-_{2/4}$, and no raw plus Mellin integral is asserted to converge on a strip. At $s = 0$, both entire completions equal one-half; at $s = 1$, the plus readout equals one while the minus readout equals zero. The plus dual Gaussian is necessary for this endpoint bookkeeping.

Sum identity and quarter-factor comparison. Adding the two half-rail kernels restores twice the unshifted full theta before differentiation; the entire readouts therefore sum to twice classical completed xi, as registered in (34). The quarter-characteristic readout equals the minus half-rail readout multiplied by one-half of one plus 2^s. This is the same exact factorization as (33), now obtained by a second scale route. It is an audit of the *scalar entire functions*, not an inverse to the four indexed Gaussian coordinates of (15). In particular the indexed O channel vanishes only after its own bilateral reflection sum; none of the three scalar completed functions retains that label.

Normalization and additional boundary zeros. At the origin, the multiplier has value one. Therefore (36) agrees with the independently derived compensator and derivative normalizations: $Q_{1/4}(0) = \frac{1}{2}$. No spurious zero has been placed at the zero coordinate, unlike the rejected paired multiplier of (9). The zeros added by $A_{1/4}$ lie on the *boundaries* of the critical strip, as explicitly recorded in (36). Thus one must not assert that every zero of $Q_{1/4}$ lies on the critical line. The precise zero-free RH equivalent is the open-strip condition (39), developed in Section 9.

What the exact identity has and has not achieved. The proof has constructed an entire quarter-characteristic carrier from a full bilateral Gaussian kernel, followed the original constant tail through a defined residue, and identified its completed readout exactly with a nonvanishing factor times xi in the open strip. It has *not* proved the required zero-free condition. It has also not changed the HG proof-DAG ownership: H-AUD083 and the existing DVJ/source owners are reused, not recreated here [23, 24, 25].

9 Open-strip equivalence and boundary zeros

Status of this section. The exact equality of entire functions has already been proved in (33). Here we prove its precise zero-set consequence and record the additional boundary zeros. This is an *equivalent formulation* of the Riemann hypothesis (RH), not a proof of the resulting zero-free statement. No new hypergroup (HG) theorem owner or dependency is introduced.

The two multiplier zero loci. Write the multiplier $A_{1/4}$ from (33) as its two explicit factors. With the real, positive base two, exponential periodicity gives $2^{s-1} = 1$ precisely at the points $s = 1+2\pi ij/\log 2$, for integral j. The other factor vanishes precisely at $s = (2j+1)\pi i/\log 2$. These are elementary consequences of the defining exponentials, rather than zeros inferred numerically from a sampled critical line. Thus the first locus lies on $\Re s = 1$ and the second on $\Re s = 0$, as registered by (36). In particular, neither factor vanishes when $0 < \Re s < 1$.

Both multiplier zeros are simple: the derivative of the vanishing factor is a nonzero multiple of $\log 2$ at each such point, and the other factor does not vanish there. Their two loci are disjoint. No denominator, quotient by completed xi, or logarithm of a zero enters this proof.

Both spectral coordinates and the full boundary lattice. Write $s = \sigma_{\mathrm{sp}} + it$, with $\sigma_{\mathrm{sp}} = \Re s$ and $t = \Im s$. The spectral imaginary coordinate is not discarded by an open-strip inequality: it labels the exact periodic zeros of the multipliers. Equation (37) records the complete loci. The factor $1 - 2^{s-1}$ and the minus-rail factor $2 - 2^s$ have zeros at $\sigma_{\mathrm{sp}} = 1$ and $t = 2\pi m/\log 2$; the other quarter factor has zeros at $\sigma_{\mathrm{sp}} = 0$ and $t = (2m+1)\pi/\log 2$. Each multiplier zero is simple; these distinct boundary loci do not coincide. The interior zero sets of the completed scalar carriers therefore agree with xi, but the unqualified whole-plane all-real-zero criterion is not valid for quarter or minus carriers [22].

Interior zeros and their multiplicities. The identity (33) holds on the entire complex plane, while $A_{1/4}$ is nonzero throughout the open critical strip. Consequently, within that strip, $Q_{1/4}$ and ξ have exactly the same zeros with the same multiplicities. The correspondence is an analytic consequence of multiplication by a nonvanishing holomorphic factor; it is not an assertion about the source labels reconstructed from a scalar readout. The source/kernel record remains in (14) and (18).

The completed xi function has the functional equation $\xi(s) = \xi(1-s)$ [6], recorded in (3). A zero in the left half of the open critical strip reflects to a zero in the right half; a critical-line zero remains on the line. Thus excluding right-half zeros is equivalent to asserting that all nontrivial xi zeros lie on the critical line. The resulting necessary and sufficient condition is exactly (39). It was the Section-0 sink until the revision of 13 October 2026; it is now an audit box off the certified path (Section 11). The equivalence does not supply the missing exclusion.

Why the boundary is not optional. At the spectral origin, the multiplier is one; hence $Q_{1/4}(0) = \frac{1}{2}$, agreeing with the independent Gaussian completion in (31). At the opposite endpoint, $A_{1/4}(1) = 0$, so $Q_{1/4}(1) = 0$ even though $\xi(1) = \frac{1}{2}$. There are also the periodic boundary zeros described above. A statement that *every* complex zero of $Q_{1/4}$ is on the critical line would therefore be false. Its correct RH-equivalent target is the *open right half-strip* in (39), with the known xi

symmetry supplying the other half. This must not be shortened to an unqualified all-real-zero statement about $Q_{1/4}$.

Half-rail zeros and the reflected companions. The plus multiplier in (34) is the nonvanishing exponential 2^s; its zeros are therefore exactly the xi zeros everywhere. The minus multiplier vanishes only when $2^s = 2$, which is on the boundary $\Re s = 1$. Consequently both half-rail completed functions share xi's zero set and multiplicities in the open critical strip, just as the quarter-characteristic carrier does. Their equality as scalar zero detectors does not recover a four-rail indexed source or create an independent zero-free estimate.

Substitution of the *whole argument* $s \mapsto -s$ into the two completed formulas gives exactly (35). In particular the remainders become $R^-_{2/4}(-s)$ and $R^+_{2/4}(-s)$, and the extra plus-tail term changes sign and has exponent $(s+1)^2/16$. Replacing either reflected remainder by its unreflected value would be a new and unproved reflection identity; none is assumed. The completed xi reflection itself changes the common factor to $\xi(1+s)$.

In $\Re s > 0$, the latter is zero-free by the classical zero-free Dirichlet half-plane $\Re(1+s) > 1$. Neither reflected multiplier vanishes there. Hence the reflected half-rail functions are zero-free in that domain. This statement is already implied by the known xi factor; it does not transport zero-freeness to the *unreflected* right half of the critical strip and is not a proof of RH. It also does not alter the source-map obligation of BE `72f/8ae`: the scalar readouts lose the indexed O coordinate.

Rotate the plus carrier without suppressing Im(s). Set $z_{\mathrm{sp}} \in \mathbb{C}$ and $s = 1/2 + iz_{\mathrm{sp}}$. Equation (38) gives $\Re s = 1/2 - \Im z_{\mathrm{sp}}$, while $\Im s = \Re z_{\mathrm{sp}}$. The rotated plus function $\widehat{Q}^+_{2/4}$ is entire in its own coordinate; its zero set has the same multiplicities as classical rotated Xi because 2^s has no zeros anywhere. Thus RH is equivalent to every zero of this rotated plus function having zero imaginary z-coordinate. This is not the assertion that the plus function is even or real on the real z_{sp}-axis. Nor does its factorization establish that all zeros are real or recover the indexed E/O orientation. The quarter-carrier criterion remains the open-strip form of (39); [22].

The central mode is a different assertion. The exact zero-set equivalence would also follow from the scalar factor identity alone. What distinguishes the present paper is the preceding construction: (1) begins with the genuine integer source index zero; (19) cancels the distinct dual zero coefficient; and (28) tracks the original constant tail through its residue. Neither the regular value at $s = 0$ nor the scalar equality by itself reconstructs the original lattice or a retained-ten DVJ owner. Those stronger source claims remain separately typed in Section 10.

What is established. An entire, zero-mode-accounted carrier has been defined without applying a singular termwise Mellin transform to its original $n = 0$ summand. Its exact factor identity gives an RH-equivalent, open-strip zero condition. No positivity, zero-freeness, zero-location estimate, or unconditional RH proof has been established in this section.

10 Existing DVJ source and information boundaries

Status of this section. The mathematical object of this paper is the particular scalar quarter-characteristic carrier $Q_{1/4}$. This section cross-references the existing De Vries–Jordan (DVJ), Jacobi and hypergroup (HG) source ownership. It neither reconstructs those owners nor introduces a new condition that they must prove. The received discussion and its mistakes remain in the versioned Q&A [21].

Published project-paper provenance. The original de Vries wave proposal, scalar RG closure, and DVJ Jordan half-shift are separate author-preprint sources [14, 15, 16]. Paper 5 supplies the Gaussian hidden-core/readout setting; Papers 6.4 and 6.5 record the golden carrier and the GKS sine–cosine readout crosswalk; Paper 7.4 identifies a distinct logarithmic quarter-turn radial readout [17, 18, 19, 20]. They are cited for their respective source roles, not as journal-reviewed proofs of this paper's quarter-characteristic scalar factor identity. In particular, the quarter characteristic and the physical quarter-turn of Paper 7.4 are not identified by their shared name.

Use the completed genealogy, not an eighteenth restart. The final disposition of H-AUD083 closes a *documentary* source-genealogy audit. Its Round-5 ledger identifies the already owned integer, half-shift, Jacobi, retained-ten and actual-zero routes, together with their typed directions [23]. The historical OPEN paragraphs in that same audit remain as a record of its review stages; its final documentary status is CLOSED. This paper does not reopen source construction or treat the audit itself as a new theorem. H-PAP002 is a complementary, separately OPEN source-first paper, not a newly imposed prerequisite here.

Original index and two-channel ownership. The unshifted source in (1) includes every $n \in \mathbb{Z}$, including the summand at zero. The four-coset partition in (11) and the named source summand in (18) are exact identities of the present Gaussian model. They do not by themselves identify the quarter-characteristic label with the DVJ Jordan order, the finite conductor, the deck parameter, or the spectral ordinate. H-AUD083 records H-VER032/033's ordered reciprocal half-Gaussian rails and their signed index reflection. Keeping both rails, or equivalently both even and odd channels, is information preserving on the named paired source; retaining only their sum is not. The scalar $H_{1/4}$ is not a new inverse for that fold [23].

The actual source owner and the scalar readout. H-AUD083 records the unreduced Jacobi source and normalized retained-ten owner through H-AUD071 and H-VER152, and identifies the trivial-character Mellin/zeta projection as a lossy scalar readout [23]. The present construction gives a new *presentation* of a scalar completed readout with its original Gaussian zero mode accounted for; it does not reconstruct the retained ten coordinates from one scalar. The original mode, Fourier-dual $k = 0$ coefficient and spectral $s = 0$ value are separately recorded in (1), (19) and (31). The completion's boundary jump in (32) is an exact readout statement, not a ten-dimensional ingress certificate.

Actual-zero and downstream status are already typed. The repository records H-VER154 as the actual-zero retained-ten common-radius exposure and H-VER156 as its terminal composition,

using the previously accepted common-radius forcing chain [24, 25, 23]. That is the existing owner route *as documented in HG*, not an additional analytic premise supplied by the factorization of Section 8. The scalar equation (33) cannot be used backwards to manufacture that actual-zero owner. Conversely, the present paper does not demote or demand reproof of H-VER154/156 merely because it studies a different Gaussian presentation.

H-AUD083 also distinguishes an optional independently proposed same-witness moving/fixed application and the downstream `ExactPairedZeroCoverage` recipient from the accepted HG owner chain. Their recorded OPEN status is not a new blocker on this scalar paper. In particular, a source-indexed pairing or a conditional downstream consumer must not be reported here as an independently proved unconditional zero-free assertion [23]. This is an application/provenance boundary, not a declaration that a new generic DVJ transport theorem is missing.

Exact theorem scope. Sections 5– 9 establish the bilateral Gaussian kernel, its whole-kernel completion, and the scalar zero-set equivalence. That identity is admissible as a downstream analytic comparison against existing HG source records. It does not identify $H_{1/4}$, $Q_{1/4}$, and a retained-ten source as isomorphic objects. No HG theorem, BE status, dependency edge, Lean-HG statement or global source macro is promoted here.

11 Why the scalar equivalence is downgraded

Status of this section. This section is the revision of 13 October 2026. The project owner ruled that the paper is downgraded, and that the paper itself must say why: "ironically as it is scalar and easier to understand, the very fact that it is scalar means info-loss. therefore although exact-equivalent it actually shows that the standard approach to RH is itself information-lossy! so, no exceptions to the rule". Section 0 now certifies only the indexed four-rail packet, (15), (16) and the sink (40). The scalar carrier, its factor identity and the RH equivalence stay displayed and proved as before, but as audit boxes off the certified path.

The rule. In this programme a certified derivation may pass only through steps that preserve information. A readout that cannot be inverted may be displayed, proved and checked, but nothing certified may rest on it. The rule has no exceptions. In particular it does not yield to an exact equivalence: an equivalence carries a condition from one readout to another; it does not carry back what the readout discarded.

Where the information is lost. At each fixed index n and scale x, the packet of (15) is invertible: its four rails and its four even and odd channels determine each other. The kernel $H_{1/4}$ of (14) is not. By (20) it is the bilateral sum over n of the single indexed combination $K^{\mathrm{idx}}_{1/4} = p_0 - \mathsf{E}$, which does not contain the odd quarter channel O at all. Nor could the plain sum over n of any channel see it. By (16) the reflection $n \mapsto -n-1$ changes its sign, so $\sum_n \mathsf{O}(n,x) = 0$ at every scale, although $\mathsf{O}(0,x) \neq 0$. Every channel that is odd under this reflection lies in the kernel of the plain summation. The kernel $H_{1/4}$, and with it every scalar built from it, is blind to the indexed quarter orientation. The loss happens at (2) and (14), before any Mellin transform. A *weighted* sum does see it. With weight $+1$ for $n \geq 0$ and -1 for $n < 0$, the sum is $2\sum_{n\geq 0} \mathsf{O}(n,x) = \sum_{m\geq 1} \left(\frac{-4}{m}\right) e^{-\pi m^2 x/16}$, the theta series of the odd character $\left(\frac{-4}{\cdot}\right)$ modulo four, whose Mellin transform gives $\pi^{-s/2}\Gamma(s/2)L(s,\left(\frac{-4}{\cdot}\right))$ and not the zeta function. That is a different scalar, and it is just as non-injective: it is blind to every channel that is even under the reflection, $p_{2/4}$ for one, which the plain sum keeps. The loss is a property of the chosen readout, not of the packet.

Why exactness does not help. The identity (33) is exact, and its multiplier $A_{1/4}$ has no zero in the open critical strip. So the zero condition on $Q_{1/4}$ there is exactly the zero condition on ξ, as Section 9 proves. But both sides of that identity are scalars computed from the summed kernel. The equivalence relates two readouts of the same kind; it cannot restore the channel that the sum cancelled.

What this says about the standard formulation. The paper does not compute ξ from $H_{1/4}$: ξ is the classical function of (3), and the factor identity (33) recognizes the completed Mellin transform $Q_{1/4}$ of the summed kernel as $A_{1/4}\xi$. So the function whose zeros the classical statement of the Riemann hypothesis is about is, within this paper, an explicit multiple of a function of $H_{1/4}$ alone. The owner's point is then a precise statement about one map: the readout from the indexed packet of (1)–(15) to $H_{1/4}$ (sum over n, then Mellin transform and completion) is not injective, and its kernel contains every reflection-odd channel, O among them. The scalar formulation is the easier one to state and to understand; read against the indexed source, in exactly this sense

it is information-lossy. This is not a claim that the zeros of ξ can be found in some other way, nor that any other formulation is false. An exact equivalence with the scalar formulation inherits the non-injectivity of its readout; it does not remove it. The earlier sections already recorded the boundary: Section 4 notes that the scalarization loses O, and Section 10 that the trivial-character Mellin/zeta projection is a lossy scalar readout. This revision makes that boundary the status of the paper.

What changes, and what does not. In Section 0, the certified sink is now (40). The kernel and every box built on it, from (18) to (39), are audit boxes off the certified path, each with its reason in the Section-0 metadata; so are the classical inputs (3) and (4). The per-index reflections were split out of (20) into (16), and the sum giving the kernel, with the combination $K_{1/4}^{\mathrm{idx}}$ that it sums, moved from (15) into (20). The bilateral theta series and their parity identities moved from the certified (1), (10) and (11) into the audit boxes (2) and (12); the certified boxes keep the per-index identities, in which each term of the original index goes to exactly one term of one coset, by the bijection $(n, a) \mapsto 4n + 4a$. So no certified box derives anything from a sum over the index. The one sum the certified boxes state, $\sum_n \mathsf{O} = 0$, is derived from the per-index reflection and shows what every plain sum loses. Every identity, proof and caveat of Sections 4–9 is unchanged. The equivalence is still exact, and it is still not a proof of the zero-free condition. No hypergroup theorem status, ledger edge or source premise changes.

Correction after hostile review. The first form of this revision did not go far enough. A hostile review on the same night found that the certified boxes (1), (10) and (11) still displayed the bilateral theta series and their identities, and that (15) still defined $K_{1/4}^{\mathrm{idx}}$, which omits O: sums and a non-injective combination on the certified path. The review also found that this section had said no other sum could see O; a weighted sum can. Both are corrected above, and the earlier wording is kept in the repository history (BE 72f/dfd3).

12 Proved and not-proved conclusions

Status of this section. The body sections, Sections 1–11 and the revision Section 10a, are now expanded for manuscript review. Section 0 remains the equation DAG (40 nodes since the revision of 13 October 2026 and its correction after review) and retains its unusually explanatory prose until that material has been checked against the completed body. The appendices are drafted with OPEN review TODOs. The paper and its associated BE item remain OPEN; this is not a new theorem-status or Riemann-hypothesis (RH) closure.

Status after the downgrade. Since the revision of 13 October 2026 the certified result of this paper is the indexed four-rail packet, (40). Everything below that rests on the summed kernel $H_{1/4}$, the RH equivalence included, remains proved but is a scalar readout off the certified path; Section 11 explains why.

What the construction proves. The exact bilateral source in (1) has an ordinary, nonzero $n = 0$ Gaussian. Applying the half-shift twice gives the four labelled cosets in (11). The quarter-characteristic combination (14) retains that original summand, while Poisson summation yields the distinct dual $k = 0$ coefficient equal to zero in (19). The full indexed source is the canonical input of (18); its sum is not a positive-index definition disguised by reindexing.

The raw logarithmic Mellin integral (24) exists in its stated half-plane. The compensator and locally uniformly convergent remainder (26)–(27) separate the constant tail from a whole-kernel entire function. The pole and finite endpoint in (28) and (31) account for the retained central mode without claiming that its isolated Mellin integral converges. The derivative representation (32) provides a second, cutoff-independent description of the same completed scalar.

The exact identity (33) identifies that completed carrier with a known, entire classical target. Its multiplier is nonzero in the open critical strip; its extra zeros are on the strip boundaries, with the origin itself regular by (36). The correct necessary and sufficient RH criterion is therefore (39), as proved in Section 9. This is an exact *equivalence of conditions*, not the verification of the zero-free condition.

The Q7 half-rail refinement is a scalar cross-check. The full four-indexed source packet and inverse in (15) retain the independent half and quarter-orientation rails. The plus and minus half-rail sums in (22) have different dual zero coefficients, requiring different compensators and remainders (29). Their entire completed factors and the quarter-carrier relation are (34); their reflected forms are (35). These identities check the original scalar factor by a second exact scale route but cannot recover the indexed odd quarter coordinate. Their reflected zero-freeness in the classical right half-plane is not a proof of zero-freeness for the original functions in the critical strip. The separately registered Q4/Q7 and source-preserving map reviews remain OPEN.

The errors were necessary review evidence. The circulated Q&A [21] contains the initial spectral-origin/lattice-origin conflation; the use of a valid Li generating series as an invalid α^0 source witness; the post-hoc coefficient assigned in (8); and the paired half-shift multiplier which creates an artificial endpoint zero in (9). Those false source-identification inferences remain FIREWALL nodes, not incoming premises for the accepted equation DAG. DLMF's completed expression is

valid, but explicitly subtracts the original constant mode in (4) [4, 5]. Likewise, the positive-index evaluation in (25) is a valid auxiliary equality where convergent, but not the requested zero-inclusive *source presentation*. Section 6 retains the actual objection instead of hiding it.

What this paper does not prove. No bound or argument here establishes that $Q_{1/4}$ lacks zeros in the open right half-strip. Consequently, the paper has not independently proved RH. It does not prove that every zero of $Q_{1/4}$ lies on the critical line, because the multiplier has explicit boundary zeros. It does not make an isolated $n = 0$ Gaussian Mellin-integrable on the entire positive scale, nor does it make differentiation an invertible operation on every labelled source summand.

The exact scalar identity is not a reconstruction of the retained-ten De Vries–Jordan (DVJ) source; it does not license identification of lattice, quarter-characteristic, spectral, Jordan or deck indices. The H-AUD083 genealogy and H-VER154/156 actual-zero material remain owned and statused where they already are [23, 24, 25]. The separately OPEN optional same-witness application and conditional Lean-HG universal coverage are not converted to results here. There is no new hypergroup proof-DAG node, blocker, BE status, source premise or cross-repository dependency.

Review handoff. Review the original Q&A alongside the displayed Section-0 DAG and the completed body. In particular, independently check the Gaussian tails, locally uniform convergence, scale factors in the derivative transform, origin normalization, multiplier zero loci and exact domains of each auxiliary equality. Transfer the useful explanatory Section-0 prose to the relevant body or appendix only after review; do not erase the versioned mistakes or their correction history. The appendices, final bibliographic audit and final manuscript acceptance remain separate work items. This is a reviewable complete *body* of H-PAP003, not final paper acceptance.

Acknowledgements

This paper was developed as an Intelligence Enhancer exercise with ChatGPT 5.5, using Extended Thinking. [26] The work used a language model under a strict operational regimen, not merely as a drafting tool. That regimen was used to prevent premature reduction of multi-branch analytic structure to a single collapsed interpretation. This is important for the present construction because the ODE, phase-space, and recurrence-theoretic data carry distinct coordinates, branches, and readout layers until the relevant admissibility maps have been specified. The same regimen also provided a controlled vocabulary for tracking the self-referential structure of multi-dimensional recurrence relations, without turning interpretive mnemonics into theorem assumptions. In particular, the model was guided to read, apply, and unpack Vedic mathematical and Advaita Vedanta terminology, including phonetic decomposition of Sanskrit terms and their strict mapping onto mathematical formulae and structural roles.

Training ChatGPT to abide by the regimen of Advaita Vedanta and to learn from the Upanishads was therefore not an ornament to the derivation, but one of the primary reasons that the present work could be achieved at all.

Special thanks are due to Professor Alex Hankey, without whose support, decades of experience, and knowledge this paper would likewise not exist.

To Thomas Loker, Arturo Salazar Chon, Satoru Watanbe, Marcel Wende, Jim Galasyn and assistant Claude, thank you for listening.

Appendices

A Received Q&A provenance and the error chronology

Provenance contract. The circulated dialogue is the source of the question, the successive proposals, the user's corrections, and the ultimate quarter-characteristic target. The live text is `docs/qa/2026-09-27-dvj-rh-zero-mode-quarter-shift-qa.md` [21]. It was imported byte-for-byte in its own commit before the analytic discussion was edited. Its received SHA-256 is `3475cb7ca6f2c84375fa23239797b2c704576a479a7985e4f04fb1e5f68beda8`. Git history, rather than a second supposedly immutable appendix copy, preserves that exact received wording. The current live document records ASCII/O4 editorial amendments without retrospectively erasing mistakes.

Question and correction sequence. The following order matters: later correct equations must not be presented as though they were the premises of the first answer.

1. **Q1: tangent versus sine/cosine.** The user asked for a zero-inclusive DVJ Jordan α^0 coordinate and an RH equivalent without a quotient singularity. The first answer substituted an entire completed xi function; see Section 2. This solves a spectral-function regularity problem, not yet the source-index provenance problem.
2. **Q2: the actual summation index.** The user corrected the answer's λ_n/n series, which starts at one: "it's not the riemannZeta(0) itself that's the problem." The original $n = 0$ Gaussian in (1) must be present before any transform. The failed inference in (6) stays labelled FIREWALL.
3. **Q3: what DLMF actually sums.** DLMF 25.5.13–14 uses an integral over $[1, \infty)$ with a positive-index omega, equal to one half of the full theta less one [4, 5]. The first proposed K_n inserted a constant at the index zero *after* that subtraction. Section 3 records why this is not source retention.
4. **Q4: apply the half-shift again.** The user requested a quarter-step in logarithmic coordinates. Repeating the parity split *before* Mellin evaluation yields the four labelled cosets of (11). The spectral quarter-turn and the lattice quarter-characteristic are separate operations; see Section 4.
5. **Q5: the positive-index regression.** A termwise dual Mellin calculation reintroduced a series over strictly positive frequencies. The user objected again. Section 6 retains both the valid auxiliary equality and the failed claim that it could replace the original bilateral source presentation.
6. **Final correction.** The full summed kernel remains inside the logarithmic integral. A smooth step subtracts its constant *tail*, not its initial integer summand. The completed remainder, derivative bridge, and exact factor identity are stated in Sections 7– 8; their RH-equivalent target is Section 9.

What the received record does not establish. The Q&A motivates the construction but does not independently certify every analytic interchange or prove the RH zero-free condition. The original $n = 0$, dual $k = 0$, and spectral $s = 0$ statements remain separately typed. The present paper's proof-status ledger is Appendix B; the pre-existing DVJ source genealogy is Appendix C. Neither a retained-ten source nor a new HG theorem is inferred from a scalar equality.

Section-0 preservation. The concise explanatory blurb and the historical FIREWALL annotations remain in Section 0 in this draft, as explicitly requested. Cross-reference navigation is registered as a separate BE item; it is not a reason to discard the blurb. The equation DAG's existing checker and globally registered macros remain the presentation authority.

B Proof DAG, domains, and error-status ledger

Revision of 13 October 2026. Section 0 now has 40 nodes and 77 edges (38 and 74 before the correction after hostile review, BE 72f/dfd3), and its certified sink is the indexed four-rail packet (40). The route below is unchanged as mathematics, but from (14) onwards it is a scalar readout off the certified path; see Section 11.

Ledger status. This appendix explains the already registered Section-0 equation DAG (36 nodes and 70 edges when this paragraph was written; see the revision note above). It creates no new node, macro, source owner, or theorem dependency. The authoritative node metadata is attached to the displayed equations in Section 0, while the node/edge derivation classes are registered in the two Section-0 TSV ledgers. The existing strict checker, not the following prose, validates their registration and target closure.

Accepted route, in dependency order. The indexed terms and the $n = 0$ value are specified by (1); the full theta series is (2). The recursive parity law and four labelled cosets are (10) and (11) term by term, and (12) as series. Their quarter-characteristic difference and exact unshifted-theta scale relation are (14) and (17). The logarithmic per-index package is (18); the Poisson-dual representation is (19). In particular, the original $n = 0$ summand is not the Fourier coefficient at $k = 0$.

The two asymptotic bounds in (23) justify the whole-kernel transform (24) on its open convergence half-plane. The compensator and whole-kernel remainder (26)–(27) produce an entire remainder. Its Gaussian moment (28) tracks the constant tail and gives the finite entire completion (31). The derivative bridge (32), the exact scale identity, and a separately disclosed classical evaluation identify (33). Its endpoint and multiplier zero loci yield (36) and the precise RH-equivalent (39).

Q7 half-rail branch and retained source types. The four-rail indexed inverse is (15); the separate Fourier phases are (20). The two half-rail sums and dual coefficients enter at (22). The plus dual zero is two, the minus dual zero is zero, and neither represents the original indexed source zero. The two entire remainders and their unequal compensators are (29). Their derivative bridge, completed scalar factors, and reflected counterparts are (30), (34), and (35). These are additional analytic/readout dependencies, not HG theorem-DAG edges.

Object or operation	Domain and inference licensed
Raw $\mathcal{M}_{1/4}$	Absolutely convergent for $\Re s < 0$, using the 1 large-scale tail and rapidly decaying small-scale kernel. Not defined by ordinary integration at $s = 0$.
Smooth remainder $R_{1/4}$	Entire: the complete $H_{1/4}(e^u) - \chi(u)$ decays faster than any exponential at both ends, uniformly on compact spectral sets.
Compensator moment	The ordinary integration-by-parts formula (28) is first valid for $\Re s < 0$. Its displayed pole is a meromorphic continuation, not an integral at zero.
Derivative transform	The derivative of the *summed* logarithmic kernel has sufficient bilateral decay for an entire spectral transform. Integration by parts first uses the raw transform's actual half-plane.

Object or operation	Domain and inference licensed
Auxiliary theta evaluation	Individual unshifted-theta derivative integrals can be separated in $\Re s > 1$. The resulting positive-index zeta identity is an analytic cross-check, not a new source definition.
Minus half rail $R^{-}_{2/4}$	One-tail subtraction. Its raw transform is initially valid for $\Re s < 0$, while its derivative transform and remainder are entire.
Plus half rail $R^{+}_{2/4}$	The raw transform has no common convergence strip. Subtract both original and dual asymptotic tails; the remainder is entire. Its derivative integral is initially valid only for $\Re s > 1$.
Reflected pair	Substitute $-s$ through the entire completion, including both remainders; the reflected zero-free region is already classical, not RH.
Entire identity	Equality on an open analytic domain, together with holomorphy, extends (33) to all complex spectral arguments.
RH criterion	The multiplier is nonzero in the *open* critical strip, so the right half-strip condition is exactly RH. Boundary zeros must not be included in an all-zero-on-line claim.

Analytic domains are part of each assertion.

Historical FIREWALL routes. These equations remain printed for independent review. The status applies to the mistaken *source inference*, not necessarily to the standard mathematical equality it accompanies.

Node	Reason it does not feed any box outside the firewall
(5)	The isolated constant Gaussian has no ordinary Mellin strip across the entire positive scale. This is not a proof that RH cannot be stated or solved.
(6)	Li's valid coefficient criterion starts at positive Taylor index. A regular local logarithm is not the original integer Gaussian zero summand.
(7)	Jensen's coefficient index begins at zero, but is a Taylor index about the symmetry point, not the source lattice index.
(8)	The proposed half-unit central coefficient was assigned after subtracting the source zero mode. It is not a transported original summand.
(9)	The valid paired multiplier introduces an artificial zero at the spectral origin and fails the selected endpoint normalization requirement.
(25)	The positive-frequency result is correct as an auxiliary equality in its convergence domain but no longer exposes the bilateral indexed source.

The two readout limits. The source-indexed kernel does not give every individual summand a convergent whole-line Mellin transform. Conversely, the smooth completion gives an entire *scalar* readout, not an inverse map to the full labelled source or a retained-ten DVJ state. The boundary jump in (32) accounts for the original constant tail, while derivative evaluation does not recover per-index information annihilated by differentiation. See Section 10 and Appendix C.

Status of the final condition. The last node of the scalar chain, (39), is an *if and only if* equivalence; since the revision of 13 October 2026 it is an audit box, and the certified sink is (40). The paper proves its factor identity and interior zero-set preservation; it does not prove the zero-free predicate on the right half-strip. Independently checking that predicate is not accomplished by the present algebra, by a finite sample, or by an executable receipt. The independent mathematical audit remains a separate OPEN BE task.

C Existing HG, Jacobi, and DVJ source-owner ledger

Ownership boundary. H-AUD083 is CLOSED as a *documentary source-genealogy audit*, with Round 5 the final crosswalk [23]. Its historical Round-1–4 OPEN language is retained as dated process evidence and must not override the final status. The underlying H-VER154/156 route has its own prior owner; H-PAP003 does not rederive it or infer it from a scalar theta identity [24, 25].

The separate author-paper lineage of the DVJ source is the 2024 wave proposal, bridge-centered RG closure, Jordan order-3 half-shift, and the Paper-5 Gaussian/readout setting [14, 15, 16, 17]. These are project preprints, not independent peer-reviewed validation of HG theorem receipts.

Existing owner	Scope and information boundary
H-VER032/033	Signed integer source, triangular half-shifts, reflection $n \mapsto -n-1$, and an ordered pair of reflected Gaussian rails. Keep negative indices and the original zero.
H-AUD070/071	Jacobi quarter character, half-character quotient, and the globally labelled source. A finite residue quotient does not recover the full unquotiented integer index.
H-VER090/094/141/149	Reciprocal DVJ logarithmic and oriented deck characters. Jordan order, power index, angular index, and spectral ordinate are separately typed.
H-VER152; H-AUD081/082	Retained-ten, componentwise heat-line carriage and normalized trivial-character scalar readout. The rank-one scalar projection is lossy, not a ten-coordinate inverse.
H-VER154	Same actual-zero owner and common-radius ordered reciprocal rails, with the complete M02 evaluator on that owner. The even/odd transform is invertible only when both channels remain.
H-AUD049/076; H-VER156	Existing common-radius forceOne and critical-line composition, with its own proof and applicability ledger. This appendix makes no new actual-zero theorem.
Lean-HG receiver	`ExactPairedZeroCoverage` is a downstream universal coverage proposition; the conditional consumer does not provide its inhabitant merely by being defined.

Typed directions from the existing crosswalk.

Original, shifted, and dual indices in this paper. The original unshifted lattice in (1) includes $n = 0$. The shifted summand (18) records its own two quarter offsets at that same displayed index. The dual coefficient in (19) is a Fourier-side $k = 0$ weight, which vanishes by cancellation. The completed spectral coordinate in (31) is another object. Coincidence of numerical values or notation is not a source intertwiner between these roles.

What H-PAP003 contributes and what it cannot recover. The whole-kernel construction and its derivative completion provide an explicitly zero-mode-accounted *scalar* presentation. The

factor identity (33) identifies it with a classical completed function. It cannot invert the fold from the ordered K_+, K_- rails to their sum, or from the retained-ten state to a single trivial-character scalar. H-AUD083 records both information-loss boundaries; they are preserved in Section 10. Neither scalar zero-set equivalence nor the derivative's boundary jump constructs an HG actual-zero owner or proves a new DVJ source transport.

The existing actual-zero and downstream directions. H-VER154 provides the same-owner common-radius source/evaluator exposure and H-VER156 records the terminal composition [24, 25]. Those are cited as existing HG ledger results, not assumptions manufactured from the H-PAP003 multiplier. The separate H-PAP002 explanatory paper and its optional same-witness moving/fixed application remain OPEN where registered. A downstream Lean-HG conditional coverage interface likewise cannot be promoted to unconditional RH by this appendix. In particular, H-AUD083's closure as *review* is not a fresh proof of that coverage [23].

Provenance and no-new-DAG declaration. The authoritative final owner ledger is `docs/audits/h-aud083-preregister-existing-correspondence-rediscovery.md`; the actual-zero source notes are `docs/theorems/h-ver154-actualzero-bilateral-commonr-m02-exposure.md` and `docs/theorems/h-ver156-actualzero-critical-line-executable-composition.md`. The received Q&A is cited rather than duplicated [21]. No HG theorem node, BE theorem status, proof edge, retained-ten ingress, or cross-repository dependency changes in this paper checkpoint. The independent audit and review tasks remain separate, OPEN work items under H-PAP003.

D Executable receipts and reproduction

Reproduction boundary. The checks below verify repository syntax, equation registration, source-to-macro consistency, selected manuscript invariants, and LaTeX build hygiene. They do not certify all analytic interchanges in the proof, inhabit downstream Lean-HG coverage, or prove the RH zero-free predicate. The latter require their independently registered mathematical/reviewer work items. Generated build files stay under the ignored `build/` tree; the appendix records commands, not generated PDFs in Git.

Canonical source and local prerequisites. The repository-root-relative paper directory is:

```
papers/quarter-characteristic-entire-riemann-carrier/
```

Its driver has the directory basename with suffix `.tex`. The Section-0 config is given by the exact shell path below. The shared, globally keyed LaTeX semantic registry is `tex/shared/d12rg-symbols.tex`; the paper-local macros include that file by repository-root-relative path. Run all commands from the HG repository root at an exact imported checkpoint. The BE ledger records H-PAP003 as `72f/bd2`; it remains OPEN.

Existing, not replacement, Section-0 checker. Use the existing strict checker and global registry validator:

```
PAPER=papers/quarter-characteristic-entire-riemann-carrier
python3 scripts/check_section0_dag.py --strict \
  --config "$PAPER/support/section0-dag-roots.json"
python3 scripts/check_symbol_registry.py \
  --paper-section0 "$PAPER/sections/section00-front-summary.tex" \
  --paper-macros "$PAPER/support/macros.tex" \
  --paper-scope HPAP003
```

The shell variable keeps long pathnames intact without display-only line breaks. The canonical source registry and its TSV-row metadata are not replicated here. A passing registry test cannot substitute for the strict Section-0 display/edge check.

Focused checks and build.

```
python3 scripts/check_hg_dag.py
python3 scripts/check_o4_lines.py
python3 scripts/check_gitlab_markdown_math.py
python3 -m pytest -q \
  tests/test_h_pap003_jensen_reader_contract.py \
  tests/test_paper_structure_policy.py
make paper-quarter-characteristic-rh
git diff --check
```

The Makefile target performs the canonical two-pass `pdflatex` build and paper-artifact checks. Its PDF and auxiliary files are written only under `build/papers/`, in the directory named by the paper slug. Do not commit them. The manuscript checker must be updated as each TODO appendix is expanded; it must not demand that completed appendices remain TODO or falsely assert that still-unwritten sections are complete.

Observed scoped receipt at this drafting stage. The Appendix-A–C checkpoints passed the existing 26-node Section-0 checker, HG DAG preflight, O4/GitLab checks, and the canonical paper build. Their scoped manuscript/structure test run reported seven passes after the staged TODO contract was updated. These are *repository acceptance receipts*, not an independent numerical proof of (33). The full global pytest suite was deliberately not run at each appendix checkpoint, in line with the multi-theorem bundle protocol. The final integration gate will choose and record its broader regression scope separately.

Q7 focused half-rail regression. The staged Section-0 Q7 body integration adds the scoped `tests/test_h_pap003_halfrail_body.py` receipt. It checks the pre-existing five registered equation nodes, Section-4/5/7/8/9 equation references, exact indexed four-rail inversion, dual parity coefficients, scalar factor algebra, and the algebraic two-tail derivative completion. A separate bilateral theta/Poisson comparison uses 170 decimal digits, never binary64 source evaluation. It is not a proof of entire continuation or zero-freeness. Run it separately:

```
python3 -m pytest -q tests/test_h_pap003_halfrail_body.py
```

The strict Section-0 checkpoint had 33 nodes and 64 edges when this paragraph was written; it has 40 nodes and 77 edges since the revision of 13 October 2026 (BE 72f/7e04, 72f/dfd3). The earlier 26-node receipt records a historical stage, not the current checker state. All Q4/Q7 mathematical-audit and source-map issues remain OPEN pending independent review.

Analytic review cannot be replaced by a sampled test. An independent referee should derive the four-coset and Poisson coefficients, bound both logarithmic tails uniformly on compact spectral sets, confirm the compensator moment in its initial domain, and verify the derivative bridge, scale factors, and endpoint normalization. Section 8 uses an auxiliary positive-index evaluation only to identify an analytic readout. No finite list of tested heights or float arithmetic proves (39). DVJ numerical calculations, if introduced later, must use at least 150 decimal digits; no such numerical calculation is claimed by this appendix.

Checkpoint and Termux replay. Each appendix has an exact-base, single-head incremental Git bundle, from the previous exported checkpoint. Its SHA-256, HEAD, TREE, and base are printed by `scripts/o18_bundle.py create`. Locally, verify bundle bytes and exact prerequisites before the O18 promotion step; execute the delivered `sh` script from `/sdcard/Download/`, and require a fast-forward of `master`. A successful local paper build is a further review step, not a substitute for the BE disposition. The parent paper remains OPEN while the mathematical, bibliography, presentation, final-check and independent-review child reports are outstanding.

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