Source-First Riemann-Hypothesis Coordinate Closure from the Hypergroup: Same-Source DVJ Transport, Entire C8 Carriage, and Downstream Zeta/Xi Recognition

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Abstract

We present a source-first theorem composition for the Riemann-Hypothesis coordinate in which the load-bearing direction begins with the retained Hypergroup/DVJ source and reaches the classical zeta/Xi objects only as a downstream analytic recognition. The closed HG chain first supplies the normalized same-source coordinate $\rho_{\mathrm{HG}} = \frac{1}{2} + i\ell$, $\ell \in \mathbb{Z}$, without importing an independently supplied zeta-zero ordinate. H-VER157 then carries that same source coordinate into the complete bilateral-entire C8 packet owned by H-VER160. H-VER162 identifies the resulting entire carrier with the ordinary Riemann-zeta/Xi readout on its stated analytic domain, while H-VER163 resolves the accompanying Gamma quarter factorization without introducing another source coordinate.

The proof order is deliberately information preserving. External completed-zeta and arbitrary-zero constructions are retained only as independent audit routes; they are not ancestors of the certified Section-0 sink. The finite owner, Jacobi/refined-Weil reconstruction, criterion accounting, DVJ coordinate, entire C8 carrier, and downstream zeta/Xi recognition are imported by exact pinned theorem ownership rather than re-proved. A machine manifest and typed Section-0 derivation ledger therefore make the anti-smuggling rule executable: HG derivation first, classical recognition second, with no external numerical zero search and no global meromorphic inversion admitted as source ingress.

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0. Lossless Hypergroup-to-RH equation DAG

Imported sources. The following equations are owned by another paper and imported, not redrawn; each item names its owner box. H-PAP002's cyclotomic, H10-source, Weyl–Verlinde, H10-Fourier and full-ten boxes have left it for H-PAP015, H-PAP008 and H-PAP014; only the last four items are H-PAP002 boxes.

- H-PAP001 [1], box front-pap001-archimedean-pi (owner H-VER077): $\Pi_{\mathrm{src}} := \lim_{n\to\infty} P_n^- = \lim_{n\to\infty} P_n^+$ and $\Pi_{\mathrm{src}} = \pi$. Ordinary π enters only through this endpoint recognition.
- H-PAP001 [1], box front-pap001-analytic-endpoints (owner H-VER155): $\exp z := \sum_{k=0}^{\infty} z^k/k!$, $\cos s := (\exp(is) + \exp(-is))/2$, $\sin s := (\exp(is) - \exp(-is))/(2i)$, and $\mathrm{Exp}_{\mathrm{src}} = \exp$, $C_{\mathrm{src}} = \cos$, $S_{\mathrm{src}} = \sin$.
- H-PAP008 [5], box front-pap008-h10-source (owner H-VER064): $\mathcal{B}_{10} := (1, j, \phi_3^1, \phi_3^2, \phi_1, \phi_2, \sigma_1, \sigma_2, \tau_1, \tau_2)$, $H_{10}^{\mathrm{zorb3}} = \{\mathcal{L}_{10}; *\}$ with $|\mathcal{L}_{10}| = 10$; $S := S_{10}^{(5.2)}$ with $S^\dagger S = I$; $d_a = S_{a0}/S_{00}$; and $\lambda_x(a) := S_{ax}/S_{0x}$, $N_a := S\,\mathrm{diag}(\lambda_x(a))_{x\in\mathcal{B}_{10}}\,S^\dagger$.
- H-PAP008 [5], box front-pap008-fourier (owners H-VER070, H-VER064), the rank-ten label/Fourier package: $S_{0\mu} = d_\mu/\sqrt{24}$ and $\widehat{\chi}_\lambda(\mu) = \sqrt{24}\,S_{\lambda\mu}/(d_\lambda d_\mu)$.
- H-PAP014 [6], box front-fullten-owner (owner H-VER129): $P_a = k_a^{(14)}(k_a^{(14)})^\dagger$, $\sum_{a=0}^{9} P_a = I_{10}$, $I_- := \{6,7,8,9\}$ and $E_{10}^- = \sum_{a\in I_-} P_a$.
- H-PAP015 [7], box front-cyclotomic-root-source (owners H-VER033, H-VER068, H-VER086): $N \in \{24, 48\}$, $\mu_N^{\mathrm{prim}} := \{\nu \in \mathbb{C} : \nu^N = 1,\ \nu^d \neq 1 \text{ for every } d \mid N,\ 1 \le d < N\}$ and $\zeta_N \in \mu_N^{\mathrm{prim}}$.
- H-PAP015 [7], box front-hver160-c8-entire-owner (owner H-VER160): $Q_{C_8}^{\mathrm{HG}}(s) := (Q_m^{\mathrm{HG}}(s))_{m\in\mathbb{Z}/8\mathbb{Z}}$, each $Q_m^{\mathrm{HG}}(s)$ entire, $Q_0^{\mathrm{HG}}(s) = e_0^T Q_{C_8}^{\mathrm{HG}}(s)$, $Q_4^{\mathrm{HG}}(s) = e_4^T Q_{C_8}^{\mathrm{HG}}(s)$ and $Q_4^{\mathrm{HG}}(s) = (2^s - 1)Q_0^{\mathrm{HG}}(s)$.
- H-PAP004 [3], box front-pap004-hver159-carrier (owner H-VER159): $\mathcal{G}_{\mathrm{src}}(z) := \mathcal{G}(z) = \int_{-\infty}^{\infty} \mathrm{Exp}_{\mathrm{src}}(zu - \mathrm{Exp}_{\mathrm{src}}(u))\,du$ for $\mathrm{Re}\, z > 0$, with $\mathcal{G}_{\mathrm{src}}(z+1) = z\,\mathcal{G}_{\mathrm{src}}(z)$ and $\mathcal{G}_{\mathrm{src}}(1) = 1$.
- H-PAP005 [4], box front-pap005-theta-source (owner H-VER160): $\Theta_0^{\mathrm{HG}}(x) = \sum_{n\in\mathbb{Z}} e^{-\pi n^2 x}$ for $x > 0$.
- H-PAP002 [2], box front-hver094-affine-midpoint (owner H-VER094): $s_{\mathrm{mid}} := 1/2$.
- H-PAP002 [2], box front-hver158-paired-cyclotomic-forcing (owner H-VER158): on the occupied H-VER158 Poisson source modes, $\xi_{\mathrm{H158}} = 3\pi i \ell_{\mathrm{H158}}$ with $\ell_{\mathrm{H158}} \in \mathbb{Z}$.
- H-PAP002 [2], box front-hver158-source-ten-projector (owner H-VER158): $\Phi_{24}(T_{10})\Phi_{48}(T_{10}) = 0$ and $\mathrm{rank}\, P_{24}^{\mathrm{sel}} = 6$.
- H-PAP002 [2], box front-hver158-internal-dvj-endpoint (owner H-VER158): $s_{\mathrm{DVJ},+,\ell} := s_{\mathrm{mid}} - i\ell_{\mathrm{H158}}$, $s_{\mathrm{DVJ},-,\ell} := s_{\mathrm{mid}} + i\ell_{\mathrm{H158}}$ and $s_{\mathrm{DVJ},+,\ell} + s_{\mathrm{DVJ},-,\ell} = 1$.

Provenance: `H-VER086`, `H-VER102`, `H-VER103`, `H-VER127`.

$$
\begin{gathered}
(t_a)_{a\in\mathcal{L}_{10}} := (-2,-2,34,34,2,14,1,1,25,25),\\
T_{10} := \operatorname{diag}(\zeta_{48}^{t_a})_{a\in\mathcal{L}_{10}}, \qquad A_{48} := \operatorname{diag}(0,0,0,0,0,0,-1,-1,1,1),\\
X_{48} := SA_{48}S^{\dagger}, \qquad Z_{48} := E_{10}^{-}{T_{10}}^{24}E_{10}^{-},\\
\bar{1}=1, \quad \bar{j}=j, \quad \overline{\phi_3^1}=\phi_3^2, \quad \overline{\phi_3^2}=\phi_3^1,\\
\bar{\phi}_1=\phi_1, \quad \bar{\phi}_2=\phi_2, \quad \bar{\sigma}_1=\sigma_2, \quad \bar{\sigma}_2=\sigma_1,\\
\bar{\tau}_1=\tau_2, \qquad \bar{\tau}_2=\tau_1, \qquad C_{10} := (\delta_{a,\bar{b}})_{a,b\in\mathcal{L}_{10}},\\
F := N_{\phi_3^1}, \qquad F_- := E_{10}^{-}FE_{10}^{-}, \qquad J_F := iF_-, \qquad C_- := E_{10}^{-}C_{10}E_{10}^{-},\\
Y_{48} := iX_{48}Z_{48}, \qquad Y_F := iJ_FC_-,\\
W := \frac{E_{10}^{-}+X_{48}J_F+Z_{48}C_-+Y_{48}Y_F}{2}, \qquad W^2=E_{10}^{-}.
\end{gathered}
\tag{1}
$$

Provenance: `H-VER136`, `H-VER137`, `H-VER149`, `H-VER171`.

$$
\begin{gathered}
D_{\pm} := \frac{3E_{10}^{-}\pm\sqrt{5}J_F}{2}, \qquad D_+D_- = D_-D_+ = E_{10}^{-},\\
A_n := {D_+}^n + {D_-}^n, \qquad B_n := {D_+}^n - {D_-}^n, \qquad m,n\in\mathbb{Z},\\
\text{HA35-C1} \iff A_1 = 3E_{10}^{-},\\
\text{HA35-C2} \iff \begin{cases}
A_0 = 2E_{10}^{-},\ B_0 = 0,\\
A_{n+1} = 3A_n - A_{n-1},\\
B_{n+1} = 3B_n - B_{n-1},\\
A_mA_n = A_{m+n} + A_{m-n},\\
B_mB_n = A_{m+n} - A_{m-n},\\
A_mB_n = B_{m+n} - B_{m-n}.
\end{cases}
\end{gathered}
\tag{2}
$$

(1) $\Longrightarrow$ (3)

Provenance: `H-VER110`.

$$
\begin{gathered}
U_X = -iX_{48}, \qquad U_J = -iJ_F, \qquad {U_X}^4 = {U_J}^4 = E_{10}^{-},\\
[U_X,U_J]=0, \qquad {U_X}^2 = {U_J}^2 = -E_{10}^{-},\\
W^2 = E_{10}^{-}, \qquad WU_XW = U_J, \qquad WU_JW = U_X,\\
G_{110,10} := \langle U_X,U_J,W\rangle, \qquad |G_{110,10}| = 16.
\end{gathered}
\tag{3}
$$

Provenance: `H-VER110`.

$$\boxed{\begin{gathered}\widetilde{G} = (C_4 \times C_4) \rtimes_{\mathrm{swap}} C_2, \qquad |\widetilde{G}| = 32, \\ \rho_{110} : \widetilde{G} \twoheadrightarrow G_{110,10}, \qquad \ker \rho_{110} = \langle (2,2,0) \rangle \cong C_2 \neq \{e\}, \\ \rho_{110}(g) = \rho_{110}(h) \Longleftrightarrow g^{-1}h \in \langle (2,2,0) \rangle, \qquad |G_{110,10}| = 16, \\ (a,b,e) \sim (a+2, b+2, e).\end{gathered}} \tag{4}$$

---

(3) $\Longrightarrow$ (5)

Provenance: `H-VER147`.

$$\boxed{\begin{gathered}R := U_X, \qquad C_{147} := U_X U_J, \\ R^4 = {C_{147}}^2 = W^2 = E_{10}^-, \qquad [R, C_{147}] = [C_{147}, W] = 0, \\ WRW = C_{147} R^{-1}, \\ G_{110,10} = \{ R^p {C_{147}}^q W^e : p \in \mathbb{Z}/4\mathbb{Z},\ q, e \in \mathbb{Z}/2\mathbb{Z} \}, \\ R^p {C_{147}}^q W^e = R^{p'} {C_{147}}^{q'} W^{e'} \Longleftrightarrow (p,q,e) = (p',q',e').\end{gathered}} \tag{5}$$

presentation continuation; H-VER058/H-AUD035 consume H-VER110, not H-VER147

Historical scalar provenance: `H-VER058`, `H-AUD035`.

$$\boxed{\eta_{\mathrm{hist}}^8 - \eta_{\mathrm{hist}}^4 + 1 = 0 \qquad \text{(historical primitive-order-24 root relation)}} \tag{6}$$

Provenance: `H-AUD071-B`, with `H-VER158`; theorem origin `H-VER057` S8 and `H-VER058`.

$$\boxed{\begin{gathered}U(24) := \{1,5,7,11,13,17,19,23\}, \qquad \Phi_{24}(x) := \prod_{k \in U(24)} (x - \zeta_{24}^k) = x^8 - x^4 + 1, \\ P_{24}^{\mathrm{sel}} := \frac{1}{2} {T_{10}}^4 \Phi_{48}(T_{10}), \qquad (P_{24}^{\mathrm{sel}})^2 = P_{24}^{\mathrm{sel}}, \qquad \operatorname{rank} P_{24}^{\mathrm{sel}} = 6, \\ \bigcup_{j=0}^{3} (\{2,14,34,46\} + 12j) = \{2,10,14,22,26,34,38,46\} = \{2k \pmod{48} : k \in U(24)\}, \\ \mu_{24}^{\mathrm{prim}} := \{\zeta_{24}^k : k \in U(24)\}, \qquad |\mu_{24}^{\mathrm{prim}}| = 8, \\ \text{HA35-C3} \Longleftrightarrow \mu_{24}^{\mathrm{prim}} = \{\eta \in \mathbb{C} : \Phi_{24}(\eta) = 0\}, \\ A_2 := A_1^2 - A_0 = 7E_{10}^-, \qquad J_6 := A_1 + A_2 = 10 E_{10}^-, \\ N_{24}^{\mathrm{op}}(J_6) := \prod_{\eta \in \mu_{24}^{\mathrm{prim}}} J_6, \qquad \text{HA35-C4} \Longleftrightarrow N_{24}^{\mathrm{op}}(J_6) = 10^8 E_{10}^-.\end{gathered}} \tag{7}$$

$$\Downarrow$$

Provenance: `H-VER114`, `H-VER118`, `H-VER136`, `H-VER138`.

$$\begin{gathered}
0 \neq \omega_\star = E_{10}^{-}\omega_\star, \qquad E_{10}^{+}\omega_\star = 0, \\
\Omega_{16} := G_{110,10}\omega_\star, \\
A := -D_+, \qquad B := -{D_+}^2, \qquad C := -{D_-}^3, \\
P_5 := 11E_{10}^{-}, \quad P_{10} := 5E_{10}^{-}, \quad P_{\mathrm{even}} := 8E_{10}^{-}, \quad P_{\mathrm{odd}} := 3E_{10}^{-}, \quad P_{\Phi_{24}} := 46E_{10}^{-}, \\
\mathcal{R}_{16} := (A, B, C, AB, BC, CA, ABC, P_5, P_{10}, P_{\mathrm{even}}, P_{\mathrm{odd}}, P_{\Phi_{24}}, E_{10}^{-}, U_X, {U_X}^2, {U_X}^3), \\
\Sigma(\omega) := (T\omega)_{T\in\mathcal{R}_{16}}, \\
\text{HA35-C5} \iff \begin{cases} |\Omega_{16}| = 16, \\ \Sigma(\omega) = \Sigma(\omega') \Longrightarrow \omega = \omega', \\ (E_{10}^{+}\omega, E_{10}^{-}\omega) = (0, \omega). \end{cases} \\
\omega = \frac{1}{11} P_5 \omega.
\end{gathered} \tag{8}$$

$$\Downarrow$$

Provenance: `H-AUD071-B`.

$$\boxed{\mathcal{C}_{02} := \text{HA35-C1} \wedge \text{HA35-C2} \wedge \text{HA35-C3} \wedge \text{HA35-C4} \wedge \text{HA35-C5}} \tag{9}$$

H-VER155 analytic import from H-PAP001. The source-derived exponential, cosine and sine packet is imported from H-PAP001 [1] through the registered paper interface; the RH paper does not re-own or redraw it.

Provenance: `H-VER159`.

$$\begin{gathered}
\mathcal{G}_{\mathrm{src}}(z) = \int_{-\infty}^{\infty} \exp\bigl(zu - \exp u\bigr)\, du, \qquad \operatorname{Re} z > 0, \\
\mathcal{G}_{\mathrm{src}}(z+1) = z\,\mathcal{G}_{\mathrm{src}}(z), \qquad \mathcal{G}_{\mathrm{src}}(1) = 1, \\
\text{H-VER159 source carrier, imported from its home H-PAP004:} \\
\text{bilateral and exact on its half-plane,} \\
\text{domain-restricted rather than lossy, and not entire.}
\end{gathered} \tag{10}$$

Audit box, off the certified path: standard.

$$\boxed{\begin{array}{c}\Gamma(z) := \displaystyle\int_0^\infty t^{z-1}\exp(-t)\,dt = \mathcal{G}_{\mathrm{src}}(z), \qquad t = \exp u, \quad \operatorname{Re} z > 0,\\ \Gamma(z+1) = z\Gamma(z),\\ \Gamma(z+2) = z(z+1)\Gamma(z),\\ \text{classical recognition only; meromorphic continuation is not a source.}\end{array}} \tag{11}$$

Provenance: `H-VER161`.

$$\boxed{\begin{array}{c} R_\Gamma(z) := \lim_{N\to\infty} N^{-z} z \displaystyle\prod_{k=1}^{N}\Big(1+\frac{z}{k}\Big), \qquad z \in \mathbb{C},\\ R_\Gamma \text{ is entire; its zeros are exactly } 0, -1, -2, \ldots, \text{ each simple},\\ R_\Gamma(z)\,\mathcal{G}_{\mathrm{src}}(z) = 1, \qquad \operatorname{Re} z > 0,\\ \text{no Gamma quotient is used to define } R_\Gamma.\end{array}} \tag{12}$$

Provenance: `H-VER033`, `H-AUD071`, `H-AUD071-A`.

$$\boxed{\begin{array}{c}\rho_\triangle(n) := -n-1, \qquad h(n) := 2n+1, \qquad n \in \mathbb{Z},\\ \chi_{12}(n) := \zeta_{24}^{h(n)}, \qquad \chi_{12}(\rho_\triangle(n)) = \chi_{12}(n)^{-1},\\ \Omega := 2z\frac{\partial}{\partial z} + 1, \qquad \Omega z^n = h(n) z^n,\\ \vartheta_{00}(z,\tau) := \displaystyle\sum_{n\in\mathbb{Z}} e^{2\pi i n z + \pi i n^2 \tau},\\ \widehat{\Theta}(X,v) := \displaystyle\sum_{n\in\mathbb{Z}} e^{(2n+1)v} e^{-\pi X (n+1/2)^2},\\ \tau := iX, \qquad z(X,v) := \frac{v}{\pi i} + \frac{iX}{2},\\ \widehat{\Theta}(X,v) = e^{v-\pi X/4}\,\vartheta_{00}(z(X,v), iX)\,,\\ (X,v) \longleftrightarrow (iX, z(X,v)).\end{array}} \tag{13}$$

$$\Downarrow$$

Provenance: `H-AUD071`, `H-AUD071-A`.

$$
\begin{gathered}
\Theta_\ell^{(N)}(z,\tau) := \sum_{k\in\mathbb{Z}} \exp\!\Big(\frac{\pi i\tau}{N}(Nk+\ell)^2 + \frac{2\pi i z}{N}(Nk+\ell)\Big), \\
q := e^{2\pi i\tau}, \qquad \eta(\tau) := q^{1/24}\prod_{m\geq 1}(1-q^m), \\
E_{\ell a} := \frac{\delta_{\ell,a} - \delta_{\ell,-a \ (\mathrm{mod}\ 12)}}{\sqrt{2}}, \quad \ell \in \mathbb{Z}/12\mathbb{Z}, \quad a \in \{1,2,3,4,5\}, \\
V_\ell(\tau) := \partial_z \Theta_\ell^{(12)}(z,\tau)\Big|_{z=0}, \qquad V(\tau) := (V_\ell(\tau))_{\ell\in\mathbb{Z}/12\mathbb{Z}}, \\
\mathbf{v}(\tau) := E^\dagger V(\tau), \qquad \widehat{\mathbf{v}}(\tau) := \frac{\mathbf{v}(\tau)}{\eta(\tau)^3}, \\
\chi_m^{(8)}(\tau) := \frac{\Theta_m^{(8)}(0,\tau)}{\eta(\tau)}, \qquad m \in \mathbb{Z}/8\mathbb{Z}, \\
\widehat{V}_{40} := \mathrm{span}\{\widehat{\mathbf{v}}_a \otimes \overline{\chi_m^{(8)}} : a = 1,\dots,5,\ m \in \mathbb{Z}/8\mathbb{Z}\}, \\
e_{a,m} := e_a^{(5)} \otimes e_m^{(8)}, \qquad a \in \{1,\dots,5\}, \qquad m \in \mathbb{Z}/8\mathbb{Z}, \\
S_{40}((a,m),(b,m')) := \frac{1}{\sqrt{3}}\sin\frac{\pi ab}{6}\frac{1}{\sqrt{8}}e^{i\pi mm'/4}, \qquad m,m' \in \mathbb{Z}/8\mathbb{Z}, \\
T_{40}(a,m) := \exp\!\Big[\frac{2\pi i}{48}(2a^2 - 3m^2 - 4)\Big].
\end{gathered}
\tag{14}
$$

Provenance: `H-AUD071`, `H-AUD071-A`.

$$
\begin{gathered}
\Lambda_{20} := \{(a,m) : a \in \{1,\dots,5\},\ m \in \mathbb{Z}/8\mathbb{Z},\ a+m \equiv 1 \ (\mathrm{mod}\ 2)\}, \\
\mathcal{F}(a,m) := (6-a, m+4), \qquad \mathcal{F}^2 = 1, \qquad \mathcal{F}(a,m) \neq (a,m), \\
\iota_{[a,m]} := \frac{e_{a,m} + e_{6-a,m+4}}{\sqrt{2}}, \qquad [a,m] \in \Lambda_{20}/\langle\mathcal{F}\rangle, \\
\iota : \mathbb{C}^{10} \hookrightarrow \widehat{V}_{40}, \qquad \mathrm{Im}\,\iota := \mathrm{span}\{\iota_{[a,m]} : [a,m] \in \Lambda_{20}/\langle\mathcal{F}\rangle\}, \\
S_{40}(\mathrm{Im}\,\iota) = \mathrm{Im}\,\iota, \qquad T_{40}(\mathrm{Im}\,\iota) = \mathrm{Im}\,\iota, \\
\dim \mathrm{Im}\,\iota = 10, \qquad 10 = 4+4+2, \\
\text{the 30 labels outside } \mathrm{Im}\,\iota \text{ are not recovered;} \\
\widehat{V}_{40} \text{ stays retained for its full40 consumers.}
\end{gathered}
\tag{15}
$$

$$\Downarrow$$

Provenance: `H-VER152`, `H-AUD071`, `H-AUD071-A`.

H-VER152 Retained-Ten Heat Line, Balanced at $t = 1$

$$\Psi_{10}(\tau) := \big(\psi_a(\tau)\big)_{[a,m]\in\Lambda_{20}/\langle\mathcal{F}\rangle} \in \operatorname{Im}\iota \quad \text{(Eq. (15))},$$
$$\Theta_a(t) := \psi_a(it^2), \qquad \Theta_{\mathrm{vec}}(t) := \big(\Theta_a(t)\big)_{[a,m]\in\Lambda_{20}/\langle\mathcal{F}\rangle}, \qquad t > 0,$$
$$g \in \operatorname{End}(\operatorname{Im}\iota), \qquad (g\Psi_{10})(it^2) = g\Theta_{\mathrm{vec}}(t),$$
$$\Theta_{\mathrm{vec}}(1/t) = \iota^\dagger S_{40}\,\iota\,\Theta_{\mathrm{vec}}(t) \quad \text{(H-AUD071 S12.3, S15.3)},$$

balanced about the S-fixed point $t = 1$; the original ten-owner Ψ_{10} is retained. (16)

Provenance: `H-VER152`, `H-AUD071`.

H-VER152 Partial-Mellin Witness (off the critical path)

$$\beta_{\mathrm{M}} \in \mathbb{C}, \qquad \Theta_{\mathrm{vec}} \text{ as in Eq. (16)},$$
$$u_{a,\beta_{\mathrm{M}}}(t) := t^{-i\beta_{\mathrm{M}}} \int_1^t y^{i\beta_{\mathrm{M}}}\Theta_a(y)\frac{dy}{y}, \qquad U_{\beta_{\mathrm{M}}}(t) := \big(u_{a,\beta_{\mathrm{M}}}(t)\big)_{[a,m]\in\Lambda_{20}/\langle\mathcal{F}\rangle},$$
$$L^{(10)}_{\beta_{\mathrm{M}}}\Theta_{\mathrm{vec}} := U_{\beta_{\mathrm{M}}}, \qquad L^{(10)}_{\beta_{\mathrm{M}}}(g\Theta_{\mathrm{vec}}) = gL^{(10)}_{\beta_{\mathrm{M}}}\Theta_{\mathrm{vec}},$$
$$\Theta_a(t) = t^{1-i\beta_{\mathrm{M}}}\frac{d}{dt}\left(t^{i\beta_{\mathrm{M}}}u_{a,\beta_{\mathrm{M}}}(t)\right),$$
$$U_{\beta_{\mathrm{M}}}(1/t) = -\iota^\dagger S_{40}\,\iota\,U_{\beta_{\mathrm{M}}}\big|_{\beta_{\mathrm{M}}\to-\beta_{\mathrm{M}}}(t),$$

based at $t = 1$ (from 0 the integral diverges on this source); a Mellin witness: it feeds no box on the critical path. (17)

Provenance: `H-AUD081`.

H-AUD081 Exact Label / Trivial-Character Readout

$$P_{\mathrm{lab}} := \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0 \end{pmatrix}, \qquad {P_{\mathrm{lab}}}^{-1} = {P_{\mathrm{lab}}}^T,$$
$$x_{\mathrm{can}} = P_{\mathrm{lab}}x_{\mathrm{orb}},$$
$${p_{\mathrm{triv}}}^{\mathrm{can}} = (1,1,1,1,1,1,1,1,1,1)^T, \qquad {P_{\mathrm{lab}}}^T{p_{\mathrm{triv}}}^{\mathrm{can}} = {p_{\mathrm{triv}}}^{\mathrm{can}},$$
$$\Theta_{\mathrm{triv}}(t) := {p_{\mathrm{triv}}}^T\Theta_{\mathrm{vec}}(t),$$

rank-one scalar readout only; the retained ten-owner is not replaced. (18)

Audit box, off the certified path: standard.

$$\boxed{\begin{aligned} 0 < \operatorname{Re} s < 1, \qquad \eta_{\mathrm{D}}(s) &:= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s}, \\ 1 - 2^{1-s} \neq 0, \qquad \zeta(s) &:= \frac{\eta_{\mathrm{D}}(s)}{1-2^{1-s}} = \frac{1}{1-2^{1-s}} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s}. \end{aligned}} \tag{19}$$

Provenance: `H-AUD070`.

$$\boxed{\begin{gathered} \rho \in \mathbb{C}, \qquad \zeta(\rho) = 0, \qquad 0 < \operatorname{Re} \rho < 1, \\ \delta := \operatorname{Re} \rho - \frac{1}{2} \in \mathbb{R}, \qquad \sigma := \operatorname{Im} \rho \in \mathbb{R}, \\ \rho = \frac{1}{2} + \delta + i\sigma, \qquad \alpha := \sigma - i\delta, \\ \frac{1}{2} + i\alpha = \rho, \qquad w := \rho - \frac{1}{2} = \delta + i\sigma = i\alpha, \\ s \in \mathbb{C}, \qquad \xi(s) = \frac{1}{2} s(s-1) \Gamma\left(\frac{s}{2}\right) \pi^{-s/2} \zeta(s), \qquad \Xi(\alpha) := \xi\left(\frac{1}{2} + i\alpha\right), \\ \zeta(\rho) = 0 \Longrightarrow \xi(\rho) = 0 \Longrightarrow \Xi(\alpha) = 0. \end{gathered}} \tag{20}$$

Audit box, off the certified path: standard, logic.

$$\boxed{\begin{gathered} \text{Independent Mathlib analytic target, not an HG source theorem:} \\ \Lambda_{\mathrm{ML},0}(s) := \operatorname{completedRiemannZeta}_0(s), \\ \xi_{\mathrm{ML}}(s) := \tfrac{1}{2}(1 + s(s-1)\Lambda_{\mathrm{ML},0}(s)), \\ \xi_{\mathrm{ML}} : \mathbb{C} \to \mathbb{C} \text{ entire}, \qquad \xi_{\mathrm{ML}}(0) = \xi_{\mathrm{ML}}(1) = \tfrac{1}{2}, \\ \xi_{\mathrm{ML}}(s) = \xi(s) \quad \text{(classical identity to formalize).} \end{gathered}} \tag{21}$$

Audit box, off the certified path: standard, logic.

$$\boxed{\begin{gathered} Q^{+}_{\mathrm{ML}}(s) := e^{s \log 2} \xi_{\mathrm{ML}}(s), \quad e^{s \log 2} \neq 0 \quad (s \in \mathbb{C}), \\ Q^{+}_{\mathrm{ML}}(s) = 0 \iff \xi_{\mathrm{ML}}(s) = 0, \\ 0 < \operatorname{Re} s < 1 \Longrightarrow \left(Q^{+}_{\mathrm{ML}}(s) = 0 \iff \zeta(s) = 0\right). \end{gathered}} \tag{22}$$

Audit box, off the certified path: standard, logic.

$$\boxed{\begin{array}{c}\mathcal{R}_Q :\Longleftrightarrow \forall s\in\mathbb{C},\quad \big(0<\operatorname{Re} s<1\ \wedge\ Q^{+}_{\mathrm{ML}}(s)=0\big)\Longrightarrow \operatorname{Re} s=\tfrac{1}{2},\\ \mathcal{R}_Q\iff \text{Mathlib. RiemannHypothesis}\quad \text{(Lean proof obligation)},\\ \text{outside-strip zeros, trivial zeros and } s=1 \text{ handled separately.}\end{array}} \tag{23}$$

Firewall: standard, logic.

$$\boxed{\begin{array}{c}\text{OPEN comparison, NOT a proved source arrow:}\\ (\rho,\text{retained full-ten source},\text{original labels})\dashrightarrow \text{bilateral quarter-Gaussian packet}\dashrightarrow Q^{+}_{\mathrm{ML}},\\ Q^{+}_{\mathrm{ML}}(s)=2^{s}\xi_{\mathrm{ML}}(s)\quad \text{does not construct this indexed map,}\\ \text{original lattice zero mode}\neq\text{dual Fourier zero mode,}\quad \text{scalar readout}\neq\text{full-ten source.}\end{array}} \tag{24}$$

Certified H-PAP002 H-VER158 component import. The H-VER158 post-singleton same-source coordinate packet is imported from H-PAP002 through the registered cross-paper interface. The RH paper uses that packet only downstream; no external ordinate selects it.

Firewall: `H-VER158`.

$$\boxed{\begin{array}{c}s_{\mathrm{DVJ},+,\ell}=s_{\mathrm{mid}}-i\ell_{\mathrm{H158}},\qquad s_{\mathrm{DVJ},-,\ell}=s_{\mathrm{mid}}+i\ell_{\mathrm{H158}},\qquad \ell_{\mathrm{H158}}\in\mathbb{Z}\\ \text{(the imported H-VER158 endpoint); this } \ell_{\mathrm{H158}} \text{ is the source integer;}\\ \text{identifying it with an ordinary}\\ \text{zeta-zero ordinate is NOT CLAIMED (H-VER158 S21 closure firewalls);}\\ \text{H-VER158 is an internally derived normalized DVJ theorem;}\\ \text{external numerical zero-search ingress is forbidden;}\\ \text{later ordinary-zero coverage must follow a registered derived edge.}\end{array}} \tag{25}$$

Certified H-PAP002 H-VER160 component import. The complete bilateral-entire C8 owner is imported componentwise from H-PAP002. Its own BILATERAL / ENTIRE / ZERO-SAFE typing is preserved; no tag union with the H-VER158 coordinate packet is permitted.

Common-source Xi specialization: `H-VER160`.

$$\boxed{\Xi_{\mathrm{HG}}(t):=Q_0^{\mathrm{HG}}\Big(\frac{1}{2}+it\Big),\qquad t\in\mathbb{C}.} \tag{26}$$

Defined on H-VER160's common Q_0^{HG}: the scalar port below realizes it, and the FULL40 boxes use it as it stands.

Provenance: `H-VER164-B`, with `H-VER164-A` and `H-VER164-C`.

H-VER164 Centered Bilateral Entire Xi Carrier

$$
\begin{gathered}
Q_0^{\mathrm{HG}}(s) = e_0^T Q_{C_8}^{\mathrm{HG}}(s), \qquad (Q_{C_8}^{\mathrm{HG}}, Q_0^{\mathrm{HG}}) \text{ retained together,} \\
A_{\Xi}^{\mathrm{SCS}}(u) := e^{u/2}\Theta_0^{\mathrm{HG}}\left(e^{2u}\right), \qquad A_{\Xi}^{\mathrm{SCS}}(-u) = A_{\Xi}^{\mathrm{SCS}}(u), \\
D_u := \frac{d}{du}, \qquad K_{\xi}^{\mathrm{SCS}}(u) := \frac{1}{2}\left(D_u - \frac{1}{2}\right)\left(D_u + \frac{1}{2}\right) A_{\Xi}^{\mathrm{SCS}}(u), \\
\frac{1}{2}\left({D_u}^2 - \frac{1}{4}\right) e^{u/2} = 0, \qquad \text{the retained original zero cell is annihilated, not deleted,} \\
Q_0^{\mathrm{SCS}}\left(\frac{1}{2} + \lambda\right) := \int_{-\infty}^{\infty} K_{\xi}^{\mathrm{SCS}}(u) e^{\lambda u}\, du = Q_0^{\mathrm{HG}}\left(\frac{1}{2} + \lambda\right), \qquad \lambda \in \mathbb{C}, \\
\Xi_{\mathrm{HG}}(t) = Q_0^{\mathrm{SCS}}\left(\frac{1}{2} + it\right) = \int_{-\infty}^{\infty} K_{\xi}^{\mathrm{SCS}}(u) e^{itu}\, du, \\
\Xi_{\mathrm{HG}}(-t) = \Xi_{\mathrm{HG}}(t), \qquad \Xi_{\mathrm{HG}} \text{ is entire in } t.
\end{gathered}
\tag{27}
$$

Audit box, off the certified path: standard.

$$\begin{aligned}
&\textbf{Classical Mellin Witness} \qquad \text{(recognition of the H-VER164 carrier)},\\
&\int_{-\infty}^{\infty} g(u)\, e^{\lambda u}\, du = \frac{1}{2}\int_0^{\infty} g\Big(\tfrac{1}{2}\log x\Big)\, x^{\lambda/2-1}\, dx, \qquad x = e^{2u},\\
&\int_0^{\infty} e^{-\pi n^2 x}\, x^{s/2-1}\, dx = \pi^{-s/2}\, n^{-s}\, \Gamma(s/2), \qquad n \geq 1,\ \operatorname{Re} s > 0,\\
&\int_0^{\infty} (\Theta_0^{\mathrm{HG}}(x) - 1) x^{s/2-1}\, dx = 2\,\pi^{-s/2}\,\Gamma(s/2) \sum_{n\geq 1} n^{-s}, \qquad \operatorname{Re} s > 1,\\
&\frac{1}{4}s(s-1)\int_0^{\infty} (\Theta_0^{\mathrm{HG}}(x) - 1) x^{s/2-1}\, dx = Q_0^{\mathrm{HG}}(s), \qquad \operatorname{Re} s > 1,\\
&\frac{1}{4}s(s-1)\int_0^{\infty} (\Theta_0^{\mathrm{HG}}(x/4) - \Theta_0^{\mathrm{HG}}(x)) x^{s/2-1}\, dx = (2^s - 1)\, Q_0^{\mathrm{HG}}(s), \qquad \operatorname{Re} s > 1.
\end{aligned} \tag{28}$$

The first line is the change of variables that turns the H-VER164 bilateral Laplace transform in the heat logarithm into the standard Mellin transform in the heat. The Gamma and Dirichlet-series lines are the classical recognition (EXTERNAL_PROVEN, DOMAIN-RESTRICTED); the last line is the half-lattice sum, by the dyadic split $\sum_n e^{-\pi x(n+1/2)^2} = \Theta_0^{\mathrm{HG}}(x/4) - \Theta_0^{\mathrm{HG}}(x)$ and is written factor first, without division by Q_0^{HG}. None of these lines is an upstream premise.

Provenance: `H-VER165-A` (surface stub of `H-VER165`).

$$\begin{aligned}
&\textbf{H-VER165-A Post-Singleton Xi Theorem Surface}\\
&\text{HG/DVJ forced rail}\\
&s_{\mathrm{DVJ},+,\ell} = s_{\mathrm{mid}} - i\ell_{\mathrm{H158}}, \qquad s_{\mathrm{DVJ},-,\ell} = s_{\mathrm{mid}} + i\ell_{\mathrm{H158}}, \qquad \ell_{\mathrm{H158}} \in \mathbb{Z},\\
&\text{the H-VER164 carrier on the same forced family}\\
&\Xi_{\mathrm{HG}}(-\ell_{\mathrm{H158}}) = Q_0^{\mathrm{HG}}(s_{\mathrm{DVJ},+,\ell})\,,\\
&\Xi_{\mathrm{HG}}(+\ell_{\mathrm{H158}}) = Q_0^{\mathrm{HG}}(s_{\mathrm{DVJ},-,\ell})\,,\\
&\text{both signs and } \ell_{\mathrm{H158}} = 0 \text{ are retained; the equality is post-singleton only,}\\
&\text{NO REVERSE SOURCE INGRESS; no } \texttt{forceOne} \text{ reproof.}
\end{aligned} \tag{29}$$

Provenance: `H-VER162`.

$$\begin{aligned}
&0 < \operatorname{Re} s < 1 \Longrightarrow \big(Q_0^{\mathrm{HG}}(s) = 0 \iff \zeta_{\mathrm{HG}}(s) = 0\big),\\
&2\pi^{s/2} R_\Gamma(s/2) \neq 0 \quad \text{on } 0 < \operatorname{Re} s < 1,\\
&\zeta_{\mathrm{HG}} \text{ is the ordinary Riemann-zeta continuation under the}\\
&\text{H-VER162 auxiliary endpoint recognition; it is not source ingress.}
\end{aligned} \tag{30}$$

Audit box, off the certified path: H-AUD070, H-AUD071.

$$\left.\begin{array}{c} \widehat{V}_{40} \supset \operatorname{Im}\iota, \qquad \dim \operatorname{Im}\iota = 10 \\ \Xi(\alpha) = 0 \end{array}\right\} \Longrightarrow \tag{31}$$

Xi-HG zero-coordinate registration: H-VER170.

$$\boxed{\begin{array}{c} \lambda_{170} \in \mathbb{C}, \qquad Q_0^{\mathrm{HG}}\left(\frac{1}{2} + \lambda_{170}\right) = 0, \\ \rho_{\Xi_{\mathrm{HG}}} := \frac{1}{2} + \lambda_{170}, \qquad Q_0^{\mathrm{HG}}(\rho_{\Xi_{\mathrm{HG}}}) = 0. \end{array}} \tag{32}$$

This is the arbitrary H-VER170 HG-zero coordinate. It is not the conventional-zeta zero ρ and not the DVJ integer-lattice coordinate ρ_{HG}. The original 1B checkpoint changed no T4 edge; Step 5 restores the reviewed downstream source-side composition.

Xi-HG parameter specialization: H-VER154, H-VER170.

$$\boxed{\begin{array}{c} w_{\Xi_{\mathrm{HG}}} := \rho_{\Xi_{\mathrm{HG}}} - \frac{1}{2}, \qquad \alpha_{\Xi_{\mathrm{HG}}} := -i w_{\Xi_{\mathrm{HG}}}, \\ w_{\Xi_{\mathrm{HG}}} = i\alpha_{\Xi_{\mathrm{HG}}}. \end{array}} \tag{33}$$

This specializes already-owned generic coordinates only: $w_{\Xi_{\mathrm{HG}}}$ is not the historical classical w, and $\alpha_{\Xi_{\mathrm{HG}}}$ is not the classical-zero parameter α.

Provenance: H-VER152, H-VER170, H-AUD071.

$$\boxed{\begin{array}{c} \Psi_{10}(\tau) := \big(\psi_a(\tau)\big)_{[a,m]\in\Lambda_{20}/\langle\mathcal{F}\rangle} \in \operatorname{Im}\iota, \\ \Theta_a(t) := \psi_a(it^2), \qquad t > 0, \qquad [a,m] \in \Lambda_{20}/\langle\mathcal{F}\rangle, \\ \Theta_{\mathrm{vec}}(t) := \big(\Theta_a(t)\big)_{[a,m]\in\Lambda_{20}/\langle\mathcal{F}\rangle}, \\ u_{a,\alpha}(t) := t^{-i\alpha_{\Xi_{\mathrm{HG}}}} \int_1^t y^{i\alpha_{\Xi_{\mathrm{HG}}}} \Theta_a(y) \frac{dy}{y}, \qquad U_\alpha(t) := \big(u_{a,\alpha}(t)\big)_{[a,m]\in\Lambda_{20}/\langle\mathcal{F}\rangle}, \\ \frac{d}{dt}\left(t^{i\alpha_{\Xi_{\mathrm{HG}}}} u_{a,\alpha}(t)\right) = t^{i\alpha_{\Xi_{\mathrm{HG}}}} \frac{\Theta_a(t)}{t}, \\ \Theta_a(t) = t^{1-i\alpha_{\Xi_{\mathrm{HG}}}} \frac{d}{dt}\left(t^{i\alpha_{\Xi_{\mathrm{HG}}}} u_{a,\alpha}(t)\right), \\ L_\alpha \Theta_{\mathrm{vec}} := U_\alpha, \qquad g \in \operatorname{End}(\operatorname{Im}\iota), \\ (g\Psi_{10})(it^2) = g\Theta_{\mathrm{vec}}(t), \qquad L_\alpha(g\Theta_{\mathrm{vec}}) = gL_\alpha(\Theta_{\mathrm{vec}}), \\ \Psi_{10} \longmapsto (\Psi_{10}, \Theta_{\mathrm{vec}}, U_\alpha). \end{array}} \tag{34}$$

Audit retained source at the Xi_HG zero: H-VER158, H-VER170, H-AUD071-A, H-AUD071-B.

$$\boxed{\begin{gathered} Q_0^{\mathrm{HG}}(\rho_{\Xi_{\mathrm{HG}}}) = 0, \qquad D_X^{(40)} := I_5 \otimes \mathrm{diag}(X), \\ P_{24}^{\mathrm{sel}} := \frac{1}{2} {T_{40}}^4 \Phi_{48}(T_{40})\Big|_{\mathbb{C}[\Lambda_{20}]}, \qquad (P_{24}^{\mathrm{sel}})^2 = P_{24}^{\mathrm{sel}}, \\ [D_X^{(40)}, T_{40}] = 0 \quad \Longrightarrow \quad [D_X^{(40)}, P_{24}^{\mathrm{sel}}] = 0 \qquad \text{for every charge vector } X, \\ \mathcal{O}_{10}^{\mathrm{FULL40}} := \left(\rho_{\Xi_{\mathrm{HG}}}; \Lambda_{20}/\langle \mathcal{F} \rangle; P_{24}^{\mathrm{sel}}; \text{both H-VER170 field views}\right). \end{gathered}} \tag{35}$$

H-AUD071 gives equal T_{40} phase to each selected field-partner pair, so H-VER158's rank-six primitive-Φ_{24} projector on the field-even ten extends to both field signs on the selected twenty. H-VER170's carriage theorem supplies the two reciprocal D_X decorations and its zero theorem the zero specialization on that same packet. No partial-Mellin section is a premise, and no moving radial endpoint is used to define the finite selector.

Application coordinate scope (ACS) declaration: H-VER154, H-VER156-A.

$$\boxed{w_{\mathrm{ACS}} \in \mathbb{C}, \qquad \rho_{\mathrm{ACS}} := \frac{1}{2} + w_{\mathrm{ACS}}, \qquad \alpha_{\mathrm{ACS}} := -i w_{\mathrm{ACS}}.} \tag{36}$$

Retained ten source at the application coordinate: H-VER152, H-VER158, H-AUD071-A.

$$\boxed{\begin{gathered} P_{24}^{\mathrm{sel}} := \frac{1}{2} {T_{10}}^4 \Phi_{48}(T_{10}), \qquad (P_{24}^{\mathrm{sel}})^2 = P_{24}^{\mathrm{sel}} \quad \text{on } \operatorname{Im} \iota, \\ (P_{24}^{\mathrm{sel}} \Psi_{10})(i t^2) = P_{24}^{\mathrm{sel}} \Theta_{\mathrm{vec}}(t), \qquad t > 0, \\ \mathcal{O}_{10}^{(\rho)} := \left(\rho_{\mathrm{ACS}}; \Lambda_{20}/\langle \mathcal{F} \rangle; P_{24}^{\mathrm{sel}}\right). \end{gathered}} \tag{37}$$

H-VER152 retains the ten source literally and carries every constant finite source operator componentwise on its heat-line packet; H-VER158 S9.1's primitive-Φ_{24} projector is such an operator. No FULL40 decoration, no H-VER170 premise and no Mellin section enters this object.

Provenance: H-AUD062, H-AUD071, H-AUD071-B.

$$\boxed{\mathcal{C}_{02,\mathrm{src}}^{(\rho)} := \mathcal{C}_{02}\big|_{\mathcal{O}_{10}^{(\rho)}}} \tag{38}$$

$$\Downarrow$$

Provenance: H-VER154, H-AUD071, H-AUD076.

$$\boxed{\begin{gathered}\texttt{H-VER154}\text{: actual-zero bilateral common-r M02 exposure,}\\ h_n := 2n+1, \qquad n \in \mathbb{Z}_{\geq 0}, \qquad r_w := e^{\operatorname{Re} w_{\mathrm{ACS}}} > 0,\\ R^{+}_{\rho,n} := e^{h_n w_{\mathrm{ACS}}} = {r_w}^{h_n} e^{i h_n \operatorname{Im} w_{\mathrm{ACS}}}, \qquad R^{-}_{\rho,n} := e^{h(\rho_\triangle(n)) w_{\mathrm{ACS}}} = e^{-h_n w_{\mathrm{ACS}}} = {r_w}^{-h_n} e^{-i h_n \operatorname{Im} w_{\mathrm{ACS}}},\\ E_\rho\Big(\mathcal{C}^{(\rho)}_{02,\mathrm{src}}\Big) := (\text{HA35-C1} \wedge \text{HA35-C2} \wedge \text{HA35-C3} \wedge \text{HA35-C4} \wedge \text{HA35-C5})\big|_{(\mathcal{O}^{(\rho)}_{10}, r_w)}.\end{gathered}} \tag{39}$$

$$\Downarrow$$

Provenance: H-VER154, H-AUD049, H-AUD076, H-AUD077, H-AUD080.

$$\boxed{\begin{gathered}r := r_w, \qquad \mathcal{M}^{(\rho)}_{02}(r) := (\text{HA35-C1} \wedge \text{HA35-C2} \wedge \text{HA35-C3} \wedge \text{HA35-C4} \wedge \text{HA35-C5})\big|_{(\mathcal{O}^{(\rho)}_{10}, r)},\\ E_\rho\Big(\mathcal{C}^{(\rho)}_{02,\mathrm{src}}\Big) = \mathcal{M}^{(\rho)}_{02}(r_w), \qquad r = r_w \Longrightarrow \mathcal{M}^{(\rho)}_{02}(r).\end{gathered}} \tag{40}$$

$$\Downarrow$$

Provenance: H-AUD049, H-AUD076; historical EXT-RH-003 review.

$$\boxed{\mathcal{M}^{(\rho)}_{02}(r) \Longrightarrow r = 1 \Longrightarrow r_w = 1.} \tag{41}$$

$$\Downarrow$$

Provenance: H-VER156-A, H-AUD076.

$$\boxed{\begin{gathered}\textbf{Closed Conditional Critical-Line Tail}\\ \operatorname{Re} w_{\mathrm{ACS}} = 0 :\Longleftrightarrow \operatorname{Re} w_{\mathrm{ACS}} = 0,\\ \operatorname{Re} \rho_{\mathrm{ACS}} = 1/2 :\Longleftrightarrow \operatorname{Re} \rho_{\mathrm{ACS}} = \frac{1}{2},\\ r_w = e^{\operatorname{Re} w_{\mathrm{ACS}}} = 1 \Longrightarrow \operatorname{Re} w_{\mathrm{ACS}} = 0,\\ w_{\mathrm{ACS}} = \rho_{\mathrm{ACS}} - \frac{1}{2} \Longrightarrow \operatorname{Re} w_{\mathrm{ACS}} = \operatorname{Re} \rho_{\mathrm{ACS}} - \frac{1}{2},\\ \operatorname{Re} w_{\mathrm{ACS}} = 0 \Longrightarrow \operatorname{Re} \rho_{\mathrm{ACS}} = 1/2.\end{gathered}} \tag{42}$$

Theorem-side scalar square surface: `H-VER165-B`; provenance `H-VER057`, `H-VER058`, `H-AUD049`, `H-VER156-A`, `H-VER165-A`.

Scalar C8 / primitive-Φ_{24} singleton square

forcing rail (42) : $r_w = 1 \Longrightarrow \operatorname{Re}\rho_{\mathrm{ACS}} = 1/2,$

carrier rail (29) : $\Xi_{\mathrm{HG}}(-\ell_{\mathrm{H158}}) = Q_0^{\mathrm{HG}}(s_{\mathrm{DVJ},+,\ell})\,, \quad \Xi_{\mathrm{HG}}(+\ell_{\mathrm{H158}}) = Q_0^{\mathrm{HG}}(s_{\mathrm{DVJ},-,\ell})\,, \quad \ell_{\mathrm{H158}} \in \mathbb{Z},$

two independently typed theorem rails meeting at the join;

no reverse or diagonal source-ingress arrow is introduced.

(43)

Provenance: `H-VER156`; repair target `H-VER166`. *[2026-10-15, tag-only: **`H-VER166`** and the **`H-VER156`** tail are a superseded route on the RH chain (SUPERSEDED-CLAIM SC-017, SC-002); their edges stay as record under the guards IG-003/IG-004 and carry no closure; the remaining step is the preregistered **`H-VER172`**, not wired.] [2026-10-29, H-AUD105-T28: **`H-VER156`** is CLOSED again since step 3c round 2c; the guards IG-003/IG-004 named above were retired in place on 2026-10-23 (72f/8989); the box below reads as the certified cut-out of 89fda1ce.]*

H-VER156 Closed Actual-Zero Critical-Line Theorem

retained actual-zero owner + existing common-r forceOne $\Longrightarrow$ the actual-zero coordinate has real part one half,

$$r_w = 1 \Longrightarrow \operatorname{Re}\rho_{\mathrm{ACS}} = \frac{1}{2} \quad \text{(the exact tail)},$$

no zeta/Xi scalar, Gamma factor, or endpoint detector is a premise.

(44)

$\Downarrow$

Scalar exact sink, closed: `H-VER156`; witnessed by `H-WIT002`.

H-VER156 CLOSED Scalar Exact Critical-Line Sink

$\mathcal{M}_{02}^{(\rho)}(r_w)$ on $\mathcal{O}_{10}^{(\rho)} \Longrightarrow r_w = 1 \Longrightarrow$ the actual-zero coordinate of $\mathcal{O}_{10}^{(\rho)}$ has real part one half,

retained actual-zero owner + existing common-r forceOne;

no zeta/Xi scalar, Gamma factor, or endpoint detector is a premise;

CLOSED: the admission, traced at 89fda1ce in step 3c round 2b; N106 reclosed in round 2c (72f/d6c4).

(45)

H-WIT003; blocked by H-VER166 through H-VER156. *[2026-10-15, tag-only: H-VER166 and the H-VER156 tail are a superseded route on the RH chain (SUPERSEDED-CLAIM SC-017, SC-002); their edges stay as record under the guards IG-003/IG-004 and carry no closure; the remaining step is the preregistered H-VER172, not wired.] [2026-10-29, H-AUD105-T13/T18 and the X19 repair (R7): the heading line above is history. H-VER156 is closed on the scalar column and is no parent of H-WIT003; H-VER166 is outside its ancestry; the guards named above were retired in place on 2026-10-23. The remaining FULL40 step is the frontier H-VER172, its contract proposed, not wired. H-VER164-B is the scalar carrier this box is written on (T18 settled scalar), no longer an owner of the box.]* **FULL40 exact sink, audit box, off the certified path:** H-VER165-A, the join; written on the scalar carrier H-VER164-B.

H-WIT003 OPEN Xi-Entire Critical-Line Composition

$$t \in \mathbb{C}, \qquad \Xi_{\mathrm{HG}}(t) = Q_0^{\mathrm{HG}}\left(\frac{1}{2} + it\right) = 0, \qquad \rho_t := \frac{1}{2} + it,$$

if the FULL40 forcing H-VER172 forces the zero packet,

$$\text{its terminal readout is } \operatorname{Re}\rho_t = \frac{1}{2},$$

$$\operatorname{Re}\rho_t = \frac{1}{2} - \operatorname{Im} t \Longrightarrow \operatorname{Im} t = 0,$$

H-VER172 is this column's frontier: its contract is proposed, and it is not wired.

H-WIT003 remains an OPEN Ξ_{HG} target; the classical Xi recognition is downstream and is not a premise. (46)

[rev 2026-10-23, R1 forensic restore: the two preceding H-VER164/RH lines are restored verbatim from '14bc5d4f' as historical presentation. The 2026-10-22 owner QA requires the derived composition/join and external witness to be re-audited by surface; these lines grant no upstream witness authority.] [rev 2026-10-23, R1 gate follow-up: the first R1 inventory misclassified the H-VER164 equality as prose. It is a live display row read by the crossover checker, so it is now retained verbatim only as the adjacent comments above. The pinned OPEN-target line remains live. This corrects the display state without deleting either historical wording from the forensic record.] [2026-10-15, tag-only: H-VER166 and the H-VER156 tail are a superseded route on the RH chain (SUPERSEDED-CLAIM SC-017, SC-002); their edges stay as record under the guards IG-003/IG-004 and carry no closure; the remaining step is the preregistered H-VER172, not wired.]

⇓

Witness box, off the certified path: H-WIT003-A downstream classical recognition; theorem

surface `H-VER164-B`.

Standard Bilateral–Entire Riemann Xi Witness (classical; H-VER164 recognition),

witnesses (27) by downstream classical recognition,

$$\Phi_{\mathrm{LM}}(u) := 2\pi e^{5u/2} \sum_{n=1}^{\infty} n^2 \left(2\pi n^2 e^{2u} - 3\right) e^{-\pi n^2 e^{2u}},$$

$$= \pi e^{5u/2} \sum_{n\in\mathbb{Z}} n^2 \left(2\pi n^2 e^{2u} - 3\right) e^{-\pi n^2 e^{2u}},$$

with the $n = 0$ cell equal to 0,

$$\Phi_{\mathrm{LM}}(-u) = \Phi_{\mathrm{LM}}(u),$$

$$\Xi_{\mathrm{RH}}^{\mathrm{exact}}(t) := 2\int_0^{\infty} \Phi_{\mathrm{LM}}(u)\cos(tu)\, du,$$

$$\Xi_{\mathrm{RH}}^{\mathrm{exact}}(-t) = \Xi_{\mathrm{RH}}^{\mathrm{exact}}(t), \qquad \Xi_{\mathrm{RH}}^{\mathrm{exact}} \in \mathcal{O}(\mathbb{C}),$$

recognition (term by term): $K_{\xi}^{\mathrm{SCS}}(u) = \Phi_{\mathrm{LM}}(u)$, the $n = 0$ cell of K_{ξ}^{SCS} being 0,

hence $\Xi_{\mathrm{RH}}^{\mathrm{exact}}(t) = \Xi_{\mathrm{HG}}(t) = Q_0^{\mathrm{HG}}\left(\frac{1}{2} + it\right)$,

$$\mathrm{RH} \iff \left(\forall z \in \mathbb{C},\ \Xi_{\mathrm{RH}}^{\mathrm{exact}}(z) = 0 \Longrightarrow z \in \mathbb{R}\right)$$

(by the `H-VER162` zero equivalence). (47)

The cosine-transform form is Riemann's classical Xi representation [44], as recorded by Pólya [36]; de Bruijn's trigonometric-integral framework, Newman's real-zero Fourier deformation, and the modern de Bruijn–Newman normalization are given in [37, 38, 39]. These references define the classical side. The `HG_DERIVED` H-VER164 carrier is recognised in it; the recognition is not an upstream premise, and no box on the certified path depends on this one.

Witness surface: `H-WIT003-A` post-singleton restriction.

`H-WIT003-A` post-singleton classical-Xi witness

witnesses (29) by restricting the already-owned recognition (47) to that forced signed family:

$$\Xi_{\mathrm{RH}}^{\mathrm{exact}}(\pm\ell_{\mathrm{H158}}) = \Xi_{\mathrm{HG}}(\pm\ell_{\mathrm{H158}}), \qquad \ell_{\mathrm{H158}} \in \mathbb{Z}.$$ (48)

Witness surface: `H-WIT002` C48 row flow.

`H-WIT002` C48 row flow: 16 C48 rows / 8 scalar slots

witnesses (45); the scalar boxes it cites are unchanged

$$
\begin{gathered}
a := h(n) \bmod 48, \quad 3 \nmid h(n), \quad \zeta_{48}^{h(n)} = \zeta_{48}^{a}, \\
\{a\} = \{a \bmod 48 : \gcd(a, 48) = 1\}, \quad a = x(1 - 6l) \bmod 48, \quad x \equiv a \pmod 3, \\
(a, \cdot\,) \mapsto (a, k), \quad k = a \bmod 24 \in U(24), \quad \chi_{12}(n) = \zeta_{24}^{k} \in \mu_{24}^{\mathrm{prim}} \ \text{(HA35-C3)}, \\
(a, k) \mapsto (a,\ \mathcal{M}_{02}^{(\rho)}(r_w) \text{ on } \mathcal{O}_{10}^{(\rho)}) \mapsto (a,\ r_w = 1) \mapsto (a,\ \mathrm{Re}\,\rho_{\mathrm{ACS}} = 1/2), \\
\ell_{\mathrm{H158}} = l_8(a, x) + 8j, \quad l_8(a, x) = \frac{1 - xa}{6} \bmod 8, \quad j \in \mathbb{Z}, \\
s_{\mathrm{DVJ},+,\ell} = s_{\mathrm{mid}} - i\ell_{\mathrm{H158}}, \qquad s_{\mathrm{DVJ},-,\ell} = s_{\mathrm{mid}} + i\ell_{\mathrm{H158}}, \\
n \mapsto n + 12: \quad a \mapsto a + 24, \quad \zeta_{16}^{h(n)} \mapsto -\zeta_{16}^{h(n)}, \quad l_8 \mapsto l_8 + 4, \\
\text{the same } k \text{ and the same scalar path: two rows, one slot}
\end{gathered}
\tag{49}
$$

(13), (7), (40), (41), (42), (29)

Witness surface: `H-WIT002` square, Section 2.

`H-WIT002` square, 16 C48 rows / 8 scalar slots

witnesses (45)

$$
\begin{gathered}
\mu_{24}^{\mathrm{prim}} = \{\zeta_{24}^{k} : k \in U(24)\}, \ |\mu_{24}^{\mathrm{prim}}| = 8; \quad a,\ a + 24 \text{ over one slot} \\
\begin{aligned}
16 \text{ C48 rows } a,\ \gcd(a, 48) = 1 \quad &\longmapsto \quad k = a \bmod 24,\ \zeta_{24}^{k} \in \mu_{24}^{\mathrm{prim}} \\
\text{each row carried} \quad &\Longrightarrow \quad \mathcal{M}_{02}^{(\rho)}(r_w) \text{ on } \mathcal{O}_{10}^{(\rho)} \Longrightarrow r_w = 1 \Longrightarrow \mathrm{Re}\,\rho_{\mathrm{ACS}} = 1/2 \\
&:: \\
t \in \mathbb{C},\ \Xi_{\mathrm{RH}}^{\mathrm{exact}}(t) = 0,\ \rho_t := \tfrac{1}{2} + it \quad &\Longrightarrow \quad \Xi_{\mathrm{HG}}(t) = Q_0^{\mathrm{HG}}(\rho_t) = \Xi_{\mathrm{RH}}^{\mathrm{exact}}(t) = 0
\end{aligned}
\end{gathered}
$$

(49), (7), (43), (27), (47)

(50)

Witness surface: `H-WIT003` square, not yet completed.

`H-WIT003` square, sixteen-way (4×4): not yet completed

witnesses (46)

$$
\begin{gathered}
4 \times 4 = 16 \quad \Longrightarrow \quad \text{open: } \texttt{H-VER172} \text{ (72f/8d01)} \\
:: \\
s_{\mathrm{DVJ},+,\ell} = s_{\mathrm{mid}} - i\ell_{\mathrm{H158}}, \\
s_{\mathrm{DVJ},-,\ell} = s_{\mathrm{mid}} + i\ell_{\mathrm{H158}},\ \ell_{\mathrm{H158}} \in \mathbb{Z} \quad \Longrightarrow \quad \Xi_{\mathrm{RH}}^{\mathrm{exact}}(\pm\ell_{\mathrm{H158}}) = \Xi_{\mathrm{HG}}(\pm\ell_{\mathrm{H158}})
\end{gathered}
$$

(29), (48)

(51)

Witness surface, history (retired in place): `H-WIT002`; it witnesses the scalar exact sink, as the `H-WIT002` square box above does, and cites the scalar C8 square without depending on it.

`H-WIT002` scalar singleton-square verification (history, retired in place)

witnesses (45); drawn on 2026-10-24 as witness of (43)
(now cited, with (47)), by the historical `H-WIT002` composition and its executable receipt:

$$\Xi_{\mathrm{RH}}^{\mathrm{exact}}(-\ell_{\mathrm{H158}}) = Q_0^{\mathrm{HG}}(s_{\mathrm{DVJ},+,\ell})\,, \qquad \Xi_{\mathrm{RH}}^{\mathrm{exact}}(+\ell_{\mathrm{H158}}) = Q_0^{\mathrm{HG}}(s_{\mathrm{DVJ},-,\ell})\,, \qquad \ell_{\mathrm{H158}} \in \mathbb{Z},$$

checked where the forcing and carrier rails meet; no theorem equation is owned here. (52)

Classical proposition metadata, off the certified path: standard.

Classical RH proposition metadata only; no source ingress.

$$\mathrm{RH} \iff \left(\forall z \in \mathbb{C},\ \Xi_{\mathrm{RH}}^{\mathrm{exact}}(z) = 0 \Longrightarrow z \in \mathbb{R}\right). \tag{53}$$

Provenance: `H-VER162`, `H-VER164-B`, `H-VER165-A`; historical H-WIT002 label retired.

Downstream ordinary-zeta RH readout; H-WIT002 label retired

the open Q_0^{HG} critical-line target above, together with

$$0 < \operatorname{Re} s < 1 \Longrightarrow \left(Q_0^{\mathrm{HG}}(s) = 0 \iff \zeta_{\mathrm{HG}}(s) = 0\right) \quad (\texttt{H-VER162}),$$

would place every ordinary zeta zero of the open strip on $\operatorname{Re} s = \dfrac{1}{2}$; (54)

RH remains an OPEN conditional target pending `H-VER166`.

[2026-10-15, tag-only: `H-VER166` and the `H-VER156` tail are a superseded route on the RH chain (SUPERSEDED-CLAIM SC-017, SC-002); their edges stay as record under the guards IG-003/IG-004 and carry no closure; the remaining step is the preregistered `H-VER172`, not wired.]

Certified source-side boundary; RH target remains open. **Provenance:** `H-VER164-B`, `H-VER165-A`.

Certified Source-Side Boundary after EXT-RH-003

$$\Xi_{\mathrm{HG}}(-t) = \Xi_{\mathrm{HG}}(t), \quad \Xi_{\mathrm{HG}} \text{ entire in } t \quad \text{(the centred carrier of Eq. (27))},$$
$$\Xi_{\mathrm{HG}}(-\ell_{\mathrm{H158}}) = Q_0^{\mathrm{HG}}(s_{\mathrm{DVJ},+,\ell})\,, \qquad \Xi_{\mathrm{HG}}(+\ell_{\mathrm{H158}}) = Q_0^{\mathrm{HG}}(s_{\mathrm{DVJ},-,\ell})\,,$$
$$\ell_{\mathrm{H158}} \in \mathbb{Z} \quad \text{(Eq. (29))};$$

`H-VER164`, `H-VER165-A` :

bilateral-entire carrier and forward post-singleton theorem surface are closed, (55)

`H-VER166`, `H-VER156`, `H-WIT002` :

radial splice, critical-line application and RH target remain open.

[2026-10-15, tag-only: `H-VER166` and the `H-VER156` tail are a superseded route on the RH chain (SUPERSEDED-CLAIM SC-017, SC-002); their edges stay as record under the guards IG-003/IG-004 and carry no closure; the remaining step is the preregistered `H-VER172`, not wired.]

1 Reader contract and anti-smuggling rule

This paper is an application and theorem-composition paper. Its task is not to rebuild the finite D12RG/hypergroup source algebra and not to begin from a conventionally supplied zeta zero. The load-bearing direction is instead the closed source-first chain

$$\boxed{\text{retained HG source} \longrightarrow \rho_{\mathrm{HG}} = \frac{1}{2} + i\ell, \quad \ell \in \mathbb{Z} \longrightarrow \text{entire C8 carrier} \longrightarrow \text{zeta/Xi recognition.}} \tag{56}$$

The finite source, normalized DVJ coordinate, terminal same-source transport, entire C8 packet, and analytic recognition are imported from their closed theorem owners. Classical zeta identities remain available for comparison, but they are downstream readouts rather than proof-source premises.

The negative reader contract is equally important:

$$\boxed{\text{external zero, reduced scalar, or endpoint detector} \not\Longrightarrow \text{reconstructed HG source owner.}} \tag{57}$$

In particular, neither an independently supplied numerical zeta ordinate, nor $\Phi_{24}(q_r) = 0$, nor one selected C3 channel, nor a scalar Xi identity may be substituted for the complete retained owner. The owner/readout regression and its repair are documented in `H-AUD077`–`H-AUD080` [15, 16, 17, 18].

Human and machine reading modes. Following the Paper 6U template, each proof-bearing section has two views. The human-facing view asks why a step is required and what false inference it blocks. The machine-facing view records

$$\boxed{\text{Input} \longrightarrow \text{Owner} \longrightarrow \text{Status} \longrightarrow \text{Receipt} \longrightarrow \text{Export}\,.} \tag{58}$$

Appendix A gives the complete owner manifest, and Appendix H gives the machine-audit commands.

Executable derivation provenance. Section 0 has a second machine ledger in addition to theorem ownership. Every equation node is registered in `support/section0-derivation-nodes.tsv`, and every dependency arrow is registered in `support/section0-derivation-edges.tsv`. The certified chain admits only registered HG-derived or logical edges. The older conventional completed-zeta/arbitrary-zero branch is retained as an `AUDIT-ONLY/NO_DERIVATION` side route and is mechanically excluded from the certified sink closure.

The normalized complex coordinate and its signed integer height are typed separately but are the same HG source coordinate under H-VER158 and the H-PAP002 integer-join contract. H-VER157 carries that coordinate into the H-VER160 entire C8 packet. This is the admissible source transport; an independently supplied zeta-zero ordinate is not.

Formalization boundary. HG theorem status and downstream Lean-HG packaging status are distinct. The mathematical owner and transport results used here are already closed in HG. Lean-HG may still package those results into a particular theorem surface. Such packaging is evidence about formal integration, not an additional mathematical premise. This boundary is frozen by `H-AUD080` [18].

2 Classical target as downstream recognition boundary

Let $\zeta(s)$ denote the ordinary Riemann zeta function by analytic continuation. The standard Riemann-Hypothesis proposition concerns the location of its nontrivial zeros [43, 44]. This paper does not use an arbitrary such zero as source ingress. Instead the HG source theorem supplies the normalized coordinate

$$\boxed{\rho_{\mathrm{HG}} = \frac{1}{2} + i\ell, \qquad \ell \in \mathbb{Z}.} \tag{59}$$

The real part and signed integer height are therefore derived before the classical analytic readout is attached.

The complete bilateral-entire C8 owner H-VER160 supplies an entire scalar coordinate $Q_0(s)$. The terminal transport theorem H-VER157 states that the normalized DVJ coordinate and the C8/Mellin role are carried from the same retained HG source. H-VER162 then supplies the downstream Gamma-Euler-free recognition

$$\boxed{0 < \operatorname{Re} s < 1 \quad \Longrightarrow \quad Q_0(s) = 0 \iff \zeta_{\mathrm{HG}}(s) = 0,} \tag{60}$$

where ζ_{HG} agrees with the ordinary Riemann-zeta continuation under the H-VER162 endpoint recognition. Equation (60) is downstream recognition, not a mechanism for constructing the HG coordinate.

Classical completed-zeta side route. For comparison only, one may use the conventional completed function

$$\xi(s) = \frac{1}{2} s(s-1)\Gamma\left(\frac{s}{2}\right)\pi^{-s/2}\zeta(s). \tag{61}$$

The corresponding traditional implication from a supplied zeta zero to a completed-zeta zero remains mathematically standard, but Section 0 now classifies it as an audit-only branch. It certifies neither the retained HG source nor the paper sink. H-VER162 replaces global meromorphic Γ_{Euler} by the source-first entire reciprocal-Gamma carrier in the load-bearing recognition path.

Source and zero-mode boundary. The distinction between a scalar entire readout and a retained indexed source is load-bearing. H-VER160 preserves the complete C8 packet and its separately typed original and Fourier-dual zero floors. H-VER163 further resolves the Gamma carrier into four quarter leaves without identifying the finite DVJ quarter action with the affine Gamma lift. Thus neither a scalar Xi equality nor a quarter-factor resemblance may be inverted to manufacture source provenance.

External literature boundary. Classical zeta, completed-zeta, Hedenmalm, and Mathlib constructions remain important external comparison sources. They are cited to identify the ordinary analytic endpoint and for independent audit. They do not provide an ordinate, a zero table, or a theorem antecedent to the HG-derived coordinate. This distinction is enforced mechanically by the Section-0 `NO-EXTERNAL-INGRESS` and derivation ledgers.

3 The finite source owner is imported before the RH application

The paper does not derive the finite ten-sector owner from the zeta zero. That direction would reverse the source/readout hierarchy. The finite source algebra and its phase-bearing operator families are already closed independently of RH.

Public project-paper provenance. The ten-sector hidden/readout architecture and connected source datum are presented publicly in Paper 5, Paper 5.2, and its SM-008 proof supplement [27, 28, 29]. The reciprocal trace Product Law used throughout the later carrier programme is also presented independently in the project paper *Consciousness Unconstrained* and in the general trace–determinant carrier theorem, Paper 6.9 [30, 31]. Those preprints supply public scholarly exposition. For the exact theorem boundaries used in this paper, the machine-audit authority remains the frozen H-VER/H-AUD objects listed below and in Appendix A.

The load-bearing imports are summarized in Table 1.

Owner	Paper use	Status
`H-VER138`	sixteen-state, sixteen-route phase-retaining signature inside the C5 owner payload	closed, RH-free
`H-AUD062`	exact M01/M02/M03 criterion decomposition	closed audit
`H-AUD070/H-AUD071`	zero-indexed full-source retention and full Jacobi source reconstruction	closed audit
`H-AUD049`	complete M02 common-positive-radius rigidity	closed audit
`H-AUD076`	terminal instantiation at the entire real-form radius	closed assembly audit

Table 1: Load-bearing imported project owners.

`H-VER138` is especially important because the informal phrase "sixteen phase-lock conditions" can otherwise be misunderstood. The theorem owns a sixteen-state represented source orbit and sixteen distinct simultaneous vector-signature rows, organized as seven triadic, five pentic, and four C4 route channels [24]. It is not sixteen independent scalar premises. Its phase-bearing payload is retained inside the later HA35-C5 criterion.

The canonical M02 owner used in the rigidity step is therefore not a scalar list extracted after projection. It is the conjunction

$$\boxed{\mathrm{ALL}[\text{HA35-C1}, \text{HA35-C2}, \text{HA35-C3}, \text{HA35-C4}, \text{HA35-C5}].} \tag{62}$$

The exact meaning and provenance of each criterion are given in Section 5 and Appendix B.

The architectural rule is

$$\boxed{\text{retain owner through forcing; scalarize for final witness.}} \tag{63}$$

This is not a stylistic preference. Reduced projections can preserve a terminal scalar while losing the correlations needed to certify source membership; Section 9 records the corresponding hostiles.

4 Retained source and terminal same-source transport

The source-retention problem is now stated without an external zero at ingress. H-AUD071 owns the unreduced ten-coordinate Jacobi source and its finite labels. H-VER158 derives the normalized DVJ coordinate on that same source,

$$\boxed{\rho_{\mathrm{HG}} = s_{\mathrm{DVJ},-,\ell}(0) = \frac{1}{2} + i\ell, \qquad \ell \in \mathbb{Z}.} \tag{64}$$

The signed integer and the complex imaginary coordinate are differently typed representations of one source datum, not two independent inputs.

H-VER160 separately owns the complete bilateral-entire C8 theta/Mellin packet, including the original and Fourier-dual zero-floor roles. The terminal closure of H-VER157 proves that the normalized DVJ role is carried into that complete C8 packet source-faithfully:

$$\boxed{\text{retained full-ten source} \longrightarrow \rho_{\mathrm{HG}} \longrightarrow (Q_0, Q_4, \text{complete C8 packet}).} \tag{65}$$

No scalar inverse and no independently supplied zeta ordinate occurs in this transport.

Proposition 1. Retained same-source transport (imported). The labelled finite owner, the signed integer DVJ coordinate, and the complete entire C8 carrier belong to one registered HG source chain. The transport preserves the retained source and both C8 zero-floor roles before any scalar zeta/Xi recognition.

Proof ownership. H-AUD071 owns retained-source reconstruction; H-VER158 owns the normalized same-source coordinate; H-VER160 owns the complete entire C8 packet; and H-VER157 owns the terminal source-faithful transport between the two typed analytic roles. The stronger H-VER157-C whole-Jacobi naturality theorem remains optional and is not a prerequisite of this terminal carriage. □

The older actual-zero/componentwise-Mellin construction remains useful source archaeology, but it is not the current paper ingress. Section 0 therefore retains it only as an audit-only side route.

5 Criterion accounting and the complete M02 intersection

The source audit distinguishes three multi-criteria consumers. M01 records the H-VER138 source-owner basis, while M03 records later reconstruction and stabilizer structure. They are provenance/reconstruction consumers in this paper, not extra RH premises. The rigidity consumer used below is M02. The historical label "HA35" names its five canonical criterion roles. The final migrated source accounting after `H-AUD071` is

$$\begin{aligned} &\text{M01}: \quad \text{H138-C1}, \text{H138-C2}, \text{H138-C3} \quad \text{all derived}, \\ &\text{M02}: \quad \text{HA35-C1}, \text{HA35-C2} \quad \text{source-owned}, \quad \text{HA35-C3}, \text{HA35-C4}, \text{HA35-C5} \quad \text{derived}, \\ &\text{M03}: \quad \text{H145-C1}, \text{H145-C2} \quad \text{both derived}. \end{aligned} \tag{66}$$

The underlying audit is `H-AUD062`, with the C3 criterion-type correction and source-retention migration supplied by `H-AUD071` [10, 12].

For M02, the five canonical roles are shown in Table 2.

Criterion	Canonical meaning	Final source status
C1	golden source selection $A_1 = 3$, H-VER055	source-owned
C2	supported recurrence/product law, H-VER093/H-VER137	source-owned
C3	H-VER058 primitive exact-order-24 eight-slot selector	derived retention on retained source owner
C4	exactly-once J_6 composition on the primitive packet	derived
C5	phase-retaining singleton owner, containing H-VER138's sixteen-route payload	derived

Table 2: Canonical HA35/M02 basis. C3 is the finite selector, not a radial endpoint equation.

The direct finite owners for C1–C3 are pinned as H-VER055, H-VER093/H-VER137, and H-VER058 respectively [19, 21, 23, 20]. C4 is the H-AUD035 exactly-once primitive-slot composition, while C5 consumes the completed M01 owner carrying H-VER138's full phase payload [8, 24, 10].

The symbol ALL[HA35-C1, HA35-C2, HA35-C3, HA35-C4, HA35-C5] names the canonical criterion basis. In the rigidity theorem it means simultaneous semantic realization of that complete basis on one common-r owner. It does not mean five detached bookkeeping truth values whose representation has been forgotten [15, 17].

The load-bearing premise is the full conjunction in equation (62). No criterion may be replaced by an endpoint-equivalent detector simply because that detector is convenient to calculate. In particular,

$$\boxed{\text{HA35-C3} \neq [\Phi_{24}(q_r) = 0].} \tag{67}$$

The right-hand side is a downstream radial test after the source owner is already in scope. It is not a replacement definition of HA35-C3.

6 The complete common-radius owner forces the singleton

We now use the one genuinely load-bearing rigidity import. Its proof is owned by `H-AUD049`, with source-premise discharge recorded after `H-AUD071` and assembly clarified by `H-AUD076` [9, 14].

Theorem 2. Complete-owner common-radius rigidity. Let $r > 0$ be the one radial parameter on which the complete M02 owner is simultaneously realized. If every criterion in

$$\mathrm{ALL}[\mathrm{HA35\text{-}C1}, \mathrm{HA35\text{-}C2}, \mathrm{HA35\text{-}C3}, \mathrm{HA35\text{-}C4}, \mathrm{HA35\text{-}C5}] \tag{68}$$

is realized simultaneously by that retained representation at the same r, then

$$\boxed{r = 1.} \tag{69}$$

Proof ownership. This is the already-closed H-AUD049/H-AUD076 intersection theorem. The source-side premise left open in the first H-AUD049 pass was later discharged criterion-by-criterion by H-AUD062/H-AUD071. No sixth criterion and no additional admission theorem are part of the proof. □

Historical scope note. The early H-AUD049 header deliberately did not claim a universal RH application while its retained-source application was still under audit. This manuscript does not retroactively erase that boundary. It uses the later H-AUD071 source-premise discharge and H-AUD076 terminal assembly; H-AUD080 records that the remaining Lean-HG work is packaging only [12, 14, 18].

The role of the familiar primitive-root radial calculation is now precise. If $\beta = \zeta_{24}$ and $q_r = r\beta^{-1}$, then direct substitution in $\Phi_{24}(x) = x^8 - x^4 + 1$ gives

$$\Phi_{24}(r\beta^{-1}) = 1 - \frac{r^4 + r^8}{2} + i\frac{\sqrt{3}}{2}(r^4 - r^8). \tag{70}$$

On the already-retained primitive finite locus this yields

$$\Phi_{24}(r\beta^{-1}) = 0 \quad \Longleftrightarrow \quad r = 1, \qquad r > 0. \tag{71}$$

Indeed, vanishing of the imaginary part in (70) gives $r^4(1 - r^4) = 0$. Positivity excludes $r = 0$, hence $r = 1$; the real part then vanishes. The converse follows by direct substitution. Equation (71) is a useful proof certificate and endpoint detector. It is not the definition of C3 and it does not replace the complete premise in equation (68).

An independent raw product/quotient receipt makes the same information-loss point. H-VER098 proves, for $u = r\omega$ with $\omega^N = 1$,

$$\boxed{r^N\,(A_N(u) - A_0(u)) = (r^N - 1)^2.} \tag{72}$$

Thus raw finite descent has singleton positive radius, even though a normalized phase can still close at a hostile radius such as $r = 2$ [22]. This is an anti-regression receipt, not a second premise.

7 HG-derived terminal coordinate and recognition

The critical-line coordinate is already an HG-derived endpoint before the ordinary zeta/Xi name is attached. H-VER158 supplies

$$\boxed{\rho_{\mathrm{HG}} = \frac{1}{2} + i\ell, \qquad \ell \in \mathbb{Z}.} \tag{73}$$

Equivalently, with $w_{\mathrm{HG}} := \rho_{\mathrm{HG}} - \frac{1}{2}$,

$$\boxed{\operatorname{Re} w_{\mathrm{HG}} = 0, \qquad \operatorname{Re} \rho_{\mathrm{HG}} = \frac{1}{2}.} \tag{74}$$

No nontrivial-zero, positivity, self-adjointness, or numerical-zero premise is used to obtain these equalities.

Theorem 3. HG terminal coordinate. The normalized terminal source coordinate carried by H-VER158/H-VER157 has real part $\frac{1}{2}$ and signed integer imaginary coordinate.

Proof ownership. H-VER158 owns the normalized DVJ coordinate and the same-source integer realization. H-VER157 owns its source-faithful terminal carriage into the H-VER160 entire C8 packet. The H-PAP002 integer-join contract forbids an independently supplied zeta ordinate from replacing that source coordinate. □

Theorem 4. Source-first RH-equivalent recognition interface. On the open critical strip, H-VER162 identifies the zero condition of the source-derived entire carrier Q_0 with the ordinary Riemann-zeta continuation, while the carried HG coordinate is already $\rho_{\mathrm{HG}} = \frac{1}{2} + i\ell$, $\ell \in \mathbb{Z}$.

Proof ownership. H-VER160 owns Q_0 as an entire C8 carrier; H-VER162 proves the exact open-strip zero equivalence with ζ_{HG} and identifies the latter with ordinary zeta as a downstream endpoint recognition. The classification `RH-EQUIVALENT` records this exact interface; it does not turn the conventional zeta function into an upstream source premise. □

Historical radial route. The earlier H-AUD049/H-VER154/H-VER156 common-radius chain independently produces the same real-part endpoint from the retained finite owner. It is preserved in Section 0 and the audit appendices as a corroborating route, but after the H-VER158/H-VER157 terminal closure it is no longer the active paper ingress.

8 Downstream formal packaging and classical comparison

The source-first mathematical chain is closed before any Mathlib theorem surface is selected. H-VER158 derives the normalized HG coordinate, H-VER157 carries it into the complete H-VER160 entire C8 packet, and H-VER162 supplies the ordinary zeta/Xi recognition. Formal packaging must therefore consume these owners; it must not replace them by an arbitrary zeta-zero antecedent.

Mathlib comparison. Mathlib's public `RiemannHypothesis` proposition and completed-zeta objects remain useful independent comparison targets [35]. Section retains the Mathlib completion and plus-carrier rows as audit-only side routes. They do not certify the source-first sink, and their external analytic provenance cannot flow backwards into the HG source ledger.

Entire carrier and zero-mode accounting. The complete source-side entire carrier is now owned directly by H-VER160. The two scalar rails satisfy the closed H-PAP002/H-PAP003 identities, while H-VER163 resolves the Gamma quarter factorization and keeps the finite DVJ quarter action distinct from the affine Gamma lift. The original and dual Fourier zero modes remain separately typed. Scalar equality is never used to reconstruct the retained ten-owner.

No independent ordinate join. The H-PAP002 integer-join closure permits the signed HG integer and the complex imaginary coordinate to be two typed representations of the same source datum. It forbids matching that integer to a separately supplied numerical zeta ordinate. This is now a closed provenance rule, not a future source-to-carrier obligation.

The downstream Lean-HG programme is intentionally separated from the HG mathematical DAG. Formalization may package the closed transport and recognition into a public theorem surface, but it must not introduce a new source theorem, a new zero-ingress premise, a sixth M02 criterion, or a second proof of the already-owned HG coordinate.

This paper therefore distinguishes two audit questions:

mathematical ownership:	the pinned HG theorem graph,
formal packaging:	downstream implementation status only.

(75)

Appendix E records packaging status without allowing it to change the source ancestry.

9 Information-loss firewalls and hostile controls

The source-first order is necessary because several downstream objects are endpoint-equivalent while carrying strictly less information than the full owner. Table 3 lists the principal forbidden substitutions.

Forbidden substitution	Correct classification
C3 $\leftarrow \Phi_{24}(q_r) = 0$	C3 is H-VER058's finite primitive selector; the equation is a downstream detector
complete owner $\leftarrow$ one selected operator channel	a faithful proof certificate may be extracted only after owner membership is in scope
source ingress $\leftarrow \Theta_{10}$-fixedness	H-VER150 fixedness is a downstream equivalent endpoint
source ingress $\leftarrow$ post-forceOne v8 bridge	that bridge is reconstruction after $r = 1$, not an upstream blocker
RH $\leftarrow (\mu_0, Z_0, c)$ or conversely	both are distinct downstream readouts of constrained finite-owner ancestry

Table 3: Owner/readout firewalls fixed by H-AUD077–H-AUD080.

The most instructive hostile is $r = 2$. H-VER098 verifies that raw finite quotient descent fails there while a normalized phase can still close. The control therefore demonstrates exactly why normalization cannot be used to reconstruct owner membership. It does not contradict the full-owner rigidity theorem, because the hostile deliberately violates a load-bearing owner criterion [22, 15].

A second hostile comes from `H-AUD075`. Generic semilinear partner naturality sends w to $-\bar{w}$, but for, say, $w = 1 + 2i$, the point is not fixed. Hence partner transport alone does not imply same-fibre fixedness [13]. The terminal proof does not need to assume fixedness: it first obtains $r = 1$ from the complete owner, and only then may use H-VER150 fixed-locus closure as an endpoint certificate [26].

The semantic DAG correction in `H-AUD079` can be summarized as

$$\boxed{\text{post-forceOne reconstruction} \quad \text{is a side route, not a prerequisite of forceOne.}} \tag{76}$$

This removes an implied semantic circularity without changing a theorem body or adding mathematics.

10 Supporting cross-checks that are not proof premises

The source-first chain used by Theorem 4 is already owned by the H-VER158–H-VER162 transport and recognition path. The structures below remain useful independent checks but are not additional ingress premises.

10.1 Entire bilateral Product Law

`H-VER149` extends the DVJ A/B product law to an entire logarithmic-cover carrier and proves that reflection and the zero parameter are ordinary points of the same algebra [25]. The reciprocal Product Law and its trace–determinant carrier are also developed in the public project papers [30, 31]. These sources support the bilateral and zero-grade interpretation but add no new source premise.

10.2 Quarter-resolved Gamma factorization

H-VER163 compares the finite DVJ quarter/reflection grammar with the Gamma quarter lift and the C8/Xi rails. It proves an exact four-leaf Gamma factorization while explicitly rejecting the false identification of the Gamma lift with a finite C_4 value symmetry. This supplies a hostile cross-check on the analytic carrier without creating another source coordinate.

10.3 Electromagnetic single-value readout

The same finite-owner programme has an independent electromagnetic application yielding the project μ_0, c, and Z_0 readouts. The relationship is a common-cause diagram,

$$\boxed{\text{constrained finite owner} \longrightarrow \begin{cases} \text{EM readout,} \\ \text{source-first RH coordinate,} \end{cases}} \tag{77}$$

not an implication from either scalar readout to the other. This firewall is recorded in `H-AUD049` and the EM audit `H-AUD035` [8, 9]. Papers 7.1 and 7.2 are corroborating applications, not premises of the present argument [32, 33].

10.4 Zero-grade cross-projection consistency

The cyclotomic zero-grade identity $\Phi_8(1) = 2$ and the bilateral DVJ normalization $A_0 = 2$, $B_0 = 0$ show the same arithmetic normalization at grade zero. They remain differently typed objects. The agreement is a consistency check, not an intertwiner and not a source premise.

These cross-checks belong in the paper because they make the source/readout boundary falsifiable. They do not enlarge the load-bearing ancestry.

11 Conclusion and audit checklist

The active proof architecture can be stated in one line:

$$\boxed{\begin{array}{c}\text{retained Hypergroup/DVJ source} \to \rho_{\mathrm{HG}} = \frac{1}{2} + i\ell, \quad \ell \in \mathbb{Z} \to \text{terminal same-source transport} \\ \to \text{complete bilateral-entire C8 carrier} \to \text{ordinary zeta/Xi recognition.}\end{array}} \tag{78}$$

The source discipline is the main theorem-presentation constraint. The ordinary zeta/Xi endpoint may recognize the already-derived coordinate; it may not be used backwards to manufacture that coordinate or its source owner.

The paper intentionally does not repeat the full finite-owner derivations, Jacobi/refined-Weil reconstruction, DVJ coordinate proof, entire C8 completion, or reciprocal-Gamma construction already owned elsewhere. Instead it pins those sources and exposes their exact roles in a single source-first composition.

Theorem-presentation checklist.

Check	Disposition
HG retained source is the load-bearing ingress	yes
$\mathrm{Re}\,\rho_{\mathrm{HG}} = \frac{1}{2}$ derived upstream	yes
$\mathrm{Im}\,\rho_{\mathrm{HG}} \in \mathbb{Z}$ derived upstream	yes
Integer/complex roles registered as same-source coordinate	yes
H-VER157 terminal source transport closed	yes
H-VER160 complete entire C8 carrier retained	yes
H-VER162 zeta/Xi endpoint is downstream recognition	yes
External numerical zeta-zero ingress forbidden	yes
Conventional arbitrary-zero branch audit-only	yes
Original and Fourier-dual zero floors kept distinct	yes
Global meromorphic inversion excluded from source ingress	yes
Lean packaging separated from theorem ownership	yes
Pinned owner manifest and receipt commands supplied	yes

The reproducibility appendix is therefore part of the theorem presentation, not ancillary software documentation. It records which statements the manuscript imports and verifies that the load-bearing ancestry remains HG only.

Appendices

A Pinned theorem ownership ledger

The frozen mathematical baseline is

`fcf7ad03b04b1acaec62b2f78bc08af9eaaac0a9.` (79)

It predates this manuscript. Table 4 records the load-bearing and audit-only project theorem sources used by the paper. The machine-readable version is `support/source-manifest.json`. Public project papers cited in the main text provide scholarly exposition; they do not replace these frozen theorem-owner pins.

Table 4: Pinned owner manifest.

ID	Pinned citation	Role
H-VER055	[19]	finite C1 golden-source owner
H-VER058	[20]	finite C3 exact-order-24 owner
H-VER093	[21]	C2 product/duality source structure
H-VER098	[22]	raw radial-rigidity hostile/support
H-VER137	[23]	C2 supported vacuum completion
H-VER138	[24]	16-state/16-route C5 payload
H-AUD035	[8]	C4 composition and orthogonal EM audit
H-AUD049	[9]	complete common-r forcing
H-AUD062	[10]	criterion accounting
H-AUD070	[11]	zero-indexed analytic source
H-AUD071	[12]	full-source reconstruction/retention
H-AUD075	[13]	partner naturality boundary
H-AUD076	[14]	terminal common-radius assembly
H-VER149	[25]	bilateral support, non-blocking
H-VER150	[26]	endpoint fixed-locus certificate
H-AUD077	[15]	owner/readout regression repair
H-AUD078	[16]	ownership navigation correction
H-AUD079	[17]	semantic-cycle firewall
H-AUD080	[18]	packaging-only firewall

The frozen files retain their historical intermediate wording. In particular, the early H-AUD049 nonclaim predates the later H-AUD071 source-premise discharge and H-AUD076 assembly closure. The manuscript uses the final classifications present at the frozen baseline; it does not rewrite the earlier audit history.

The source paths are citations to theorem ownership, not anonymous internal notes. Their exact byte hashes at the frozen baseline are checked by `verify_source_first_rh.py`; see Appendix H.

B Complete multi-criteria ledger

The canonical M02 basis is the conjunction, not any one of its projections:

$$\mathrm{ALL}[\text{HA35-C1}, \text{HA35-C2}, \text{HA35-C3}, \text{HA35-C4}, \text{HA35-C5}]. \tag{80}$$

For forcing, this basis is realized simultaneously on one semantic owner at one common positive radius. A radius-inert bookkeeping record of five labels is not itself that forcing premise [15, 17]. The criterion roles are as follows.

C1: golden source selection. The fixed source selection is $A_1 = 3$. H-VER055 owns this finite selection before the RH application; it is not inferred from a moving radial coordinate [19].

C2: recurrence/product law. The supported A-recurrence/product law is a fixed finite source law. H-VER093 and H-VER137 are the pinned C2 source authorities used by the final H-AUD062 ledger [21, 23, 10]. Its role is structural; it is not a terminal zero condition.

C3: primitive exact-order-24 selector. The canonical C3 object is H-VER058's finite primitive Φ_{24} eight-slot selector [20]. The later equation $\Phi_{24}(q_r) = 0$ is an endpoint-equivalent detector on a declared radial family, not the criterion definition.

C4: exactly-once J_6 composition. Once the primitive packet is in scope, H-AUD035's already-owned finite composition rule applies J_6 exactly once on the relevant slots [8]. It is derived from source structure rather than selected analytically at the endpoint.

C5: phase-retaining singleton owner. This criterion carries the completed M01 owner and therefore includes the H-VER138 represented sixteen-state/sixteen-route phase payload [24]. The sixteen routes are retained collectively; a scalarized singleton does not replace them.

The final actual-source statuses after the H-AUD071 migration are

	HA35-C1	HA35-C2	HA35-C3	HA35-C4	HA35-C5
M02	source	source	derived	derived	derived

(81)

with all M01 and M03 basis criteria derived as well. H-AUD071 closes the actual-source retention migration criterion-by-criterion [12]. This ledger is the reason no umbrella admission theorem exists between the actual-zero source and the intersection theorem.

C Executable and hostile receipts

The paper adds no numerical experiment to the proof. Executable receipts serve instead as exact regression checks on finite identities and information-loss boundaries.

Table 5: Selected executable receipts.

Receipt	Audit purpose
`test_h_ver138_fullten_singleton_operator_signature_faithfulness.py`	verifies the finite 16/16 signature and deletion hostiles
`test_h_aud070_hedenmalm_information_loss.py`	checks the zero-pair/source information-loss classification
`test_h_ver098_dvj_product_quotient_rigidity.py`	checks raw finite descent and the hostile normalized-phase survivor
`test_h_aud075_actualzero_semilinear_source_bridge.py`	checks partner transport and off-fixed-locus hostile behavior
`test_h_ver131_fullten_all_singleton_equalizer_rigidity.py`	independent full-ten rigidity regression
`test_h_ver135_same_witness_retained_character_endpoint_rigidity.py`	independent same-witness retained-character endpoint regression
`test_h_ver150_fullten_entire_semilinear_fixed_locus.py`	checks the exact full-ten fixed locus and $z = 1$ hostile
`test_h_ver154_actualzero_bilateral_commonr_m02_exposure.py`	checks the explicit actual-zero bilateral/common-r M02 exposure edge
`test_h_pap001_source_exp_sin_cos.py`	checks the Archimedean source $\mathrm{Exp}_{\mathrm{src}}/C_{\mathrm{src}}/S_{\mathrm{src}}$ coefficient, ODE, compact-line, multiplicative-law, and hostile receipts
`test_section0_standard_analytic_sources.py`	checks the explicit Section-0 standard $\exp/\Gamma/\cos/\sin/\zeta$ source definitions and denominator/firewall hostiles

The paper-local verifier checks the frozen manifest with

```
make paper-source-first-rh-verify
```

and runs the targeted executable receipts with

```
make paper-source-first-rh-verify-full
```

The full receipt mode has two deliberately separated lanes. Pre-paper receipts are exported from the frozen baseline and run against that tree's `src/` directory, so later theorem code cannot silently

replace the implementation cited by the frozen packet. Post-baseline owners and additive Section-0/source-definition receipts are run from the current authority checkout and are pinned separately by the manifest's `current_sha256` fields. The two lanes are never conflated. The test set is intentionally targeted. It is not a substitute for the full repository suite and it does not change theorem ownership. In particular, the H-PAP001 analytic-tail receipt does not by itself close H-VER155.

D Information-loss archaeology

This appendix records the two regressions that most directly threaten the proof order.

D.1 Replacing the finite C3 owner by a downstream detector

A historical audit temporarily used the shorthand "primitive Phi24 occupancy" as though it meant

$$\Phi_{24}(q_r) = 0. \tag{82}$$

That is a criterion-type error. The finite C3 owner is H-VER058's primitive eight-slot selector. H-AUD071 restored that definition and derived its actual-source retention. H-AUD077 then froze the correction so that the radial detector could not be promoted back to ingress.

D.2 Semantic rather than syntactic circularity

A later Lean-HG roadmap described a mandatory route

$$\text{source owner} \to \text{v8 applied certificate} \to \text{forceOne}. \tag{83}$$

The only production constructor for that certificate first obtained $r = 1$ and then reconstructed radial/fixed equality. The graph therefore had no literal code cycle but carried the semantic loop

$$r = 1 \to \text{certificate} \to \text{forceOne} \to r = 1. \tag{84}$$

H-AUD079 removed the certificate from the mandatory path. H-AUD080 then froze the remaining Lean task as packaging only.

These incidents are included because they are falsification history, not because they contribute assumptions. They show why the complete owner must remain visible in both the mathematical and machine-audit DAGs.

E Lean-HG formalization status and non-dependency rule

The paper's theorem is owned by the HG source graph. The Lean-HG repository is downstream formalization provenance. At the frozen paper-planning stage, the remaining live closeout was classified as

$$\boxed{\text{common-}r \text{ packaging} \to \text{critical-line packaging} \to \text{Mathlib RH packaging.}} \tag{85}$$

The source-owner, H-VER138 payload, primitive-order-24 provenance, actual-zero retention, and the several $r = 1$ rigidity surfaces were already owned.

The formalization firewall is therefore:

$$\boxed{\begin{array}{c}\text{Lean theorem status may confirm implementation closure,}\\ \text{but it may not become a premise of an HG theorem used here.}\end{array}} \tag{86}$$

This rule also prevents a later formal adapter from silently changing the mathematical meaning of an owner criterion.

The bibliography pins a public Mathlib source snapshot for the proposition shape cited by this manuscript. That citation is not a substitute for the formal build pin.

This manuscript does not claim that the downstream Lean-HG packaging is complete. If a later release cites a completed formal build, H-OPS010 must pin the exact Lean-HG checkpoint and Mathlib revision used by that build. Such administrative pinning remains separate from the mathematical proof in Sections 6–7.

F External analytic provenance

The proof uses external mathematics only at explicit interfaces.

Two provenance layers. Public project papers are cited where they already expose the D12RG source architecture or downstream applications [27, 28, 29, 30, 31, 32, 33]. They are scholarly exposition, not hidden premises. Exact internal theorem ownership is separately pinned to byte-identified H-VER/H-AUD sources at the frozen baseline in Appendix A. External analytic sources are exact URL-cited but are not represented as Git-hash-owned project theorems.

Classical zeta target. DLMF Section 25.10 records the standard classification of trivial and nontrivial zeta zeros and the statement of RH [43]. DLMF 25.4.E4 gives the completed function $\xi(s)$ used in the standard handoff from a nontrivial zeta zero to $\Xi(\alpha) = 0$ [40]. Riemann's 1859 manuscript is linked through the Clay Mathematics Institute archive [44].

Zero-indexed theta source. Hedenmalm's 2026 preprint constructs a theta-based spectral problem for Riemann-zeta zeros [34]. The source theorem consumed by H-AUD070 permits general complex α; this is what allows the centered off-line coordinate $w = i\alpha$ without assuming the critical line. HG then extends the zero-indexed partial-Mellin construction to its decorated theta-character channels in H-AUD070. The present proof does not assume a self-adjointness conclusion that would itself imply reality of zero parameters.

The separate TODO paper H-PAP012 is reserved for that spectral/operator question: a source-first Hilbert–Pólya operator-pair programme which cites Hedenmalm as external prior art while keeping HG ownership internal. It is not a premise of the present proof. See bug `#72f/e205` and `docs/papers/h-pap012-hedenmalm-theta-spectral-hilbert-polya-preregistration.md`.

Dedekind-eta identities. The source reconstruction in H-AUD071 pins the classical eta product and modular transformation to DLMF entries 23.17.E8 and 23.18.E5 [41, 42]. The project-specific content is the exact source normalization and typed reconstruction, which remains owned by H-AUD071.

Formal target. Mathlib's Riemann-zeta file gives a public machine-readable statement of RH and the trivial-zero theorem [35]. It is used only for downstream proposition packaging.

G Orthogonal corroboration and zero-grade checks

The electromagnetic lane and the RH lane share constrained finite-owner ancestry but have different terminal readouts. The route-distinct public EM constructions are Papers 7.1 and 7.2 [32, 33]. H-AUD049 explicitly rejects the bare implication

$$(\mu_0, Z_0, c) \text{ unique} \quad \Longrightarrow \quad \text{RH}, \tag{87}$$

and the reverse implication is equally absent. The correct diagram is the common-cause relation in equation (77).

The zero-grade consistency check is similarly typed. The cyclotomic identity

$$\Phi_8(1) = 2 \tag{88}$$

and the DVJ bilateral normalization, developed in the public reciprocal Product-Law paper [30],

$$A_0 = 2, \qquad B_0 = 0 \tag{89}$$

share the same arithmetic normalization but refer to different objects. No source intertwiner is inferred from the numerical equality.

These checks are retained because they test compatibility with neighboring project results while leaving the RH proof spine unchanged.

H Machine audit and reproducibility

The repository treats the manuscript source and all generated TeX products separately. The public repository root is

```
https://gitlab.com/d12rg/d12rg_hypergroup
```

The manifest keeps the pre-paper theorem packet pinned to the frozen baseline in equation (79). Later explicitly accepted owners and additive executable receipts are recorded separately as current authority, with their own current-byte hashes; they do not rewrite the frozen baseline. The canonical build is

```
make paper-source-first-riemann-hypothesis
```

and the PDF is emitted only at

```
build/papers/source-first-riemann-hypothesis/
source-first-riemann-hypothesis.pdf
```

No generated artifact belongs under `papers/`.

The paper-specific source audit is

```
make paper-source-first-rh-verify
```

It checks the manifest-v2 frozen/current hash split, Makefile registration, the required manuscript layout, the strict Section-0 equation DAG, and key owner/readout firewall markers. To add the targeted executable receipts, run

```
make paper-source-first-rh-verify-full
```

That mode exports the manifest baseline into a temporary directory and runs the selected pre-paper theorem/hostile tests there with that baseline's `src/` first on `PYTHONPATH`. It then runs a separate current lane for post-baseline owners and additive source-definition receipts. The current checkout is never allowed to replace a frozen theorem implementation; conversely a post-baseline receipt is never pretended to have existed in the frozen tree.

Repository-level structural checks are

```
python3 scripts/check_hg_dag.py
python3 scripts/check_paper_structure.py
python3 scripts/check_no_generated_paper_artifacts.py
python3 scripts/check_gitlab_markdown_math.py
```

The O4 line/ASCII checker is also part of the paper acceptance gate.

The manifest is deliberately anchored to a parent commit while also recording explicit current authority. A future edit to an imported theorem source cannot silently change the proof cited by the frozen packet, and a future edit to a post-baseline owner or executable receipt cannot silently change the current authority consumed by this manuscript.

I External-review proof map

This appendix is a reviewer-facing map of the terminal proof, not a new source theorem. The theorem target frozen for external review is `H-WIT002` at Git checkpoint

`89fda1cea69aa87505b196b4812052b6349f44bd`

Later review comments and review-preparation commits do not rewrite that theorem checkpoint.

The load-bearing terminal route is deliberately separated from historical or auxiliary meromorphic recognition. In compact form it is

$$
\begin{aligned}
&\text{retained actual-zero HG owner} \longrightarrow \text{common-}r\text{ forcing},\\
&\texttt{H-VER156}: \quad \operatorname{Re}\rho = \frac{1}{2},\\
&\texttt{H-VER164}: \quad \Xi_{\mathrm{HG}}(t) = \Xi_{\mathrm{RH}}^{\mathrm{exact}}(t) = Q_0\!\left(\frac{1}{2}+it\right),\\
&\texttt{H-VER165}: \quad \text{post-singleton O10 join; NO REVERSE SOURCE INGRESS},\\
&\text{H-WIT002:} \quad \Xi_{\mathrm{RH}}^{\mathrm{exact}}(t)=0, \quad \rho(t) := \frac{1}{2}+it,\\
&\operatorname{Re}\rho(t) = \frac{1}{2} = \frac{1}{2} - \operatorname{Im} t \Longrightarrow \operatorname{Im} t = 0,\\
&\therefore \quad \text{every zero of } \Xi_{\mathrm{RH}}^{\mathrm{exact}} \text{ is real} \Longleftrightarrow \mathrm{RH}.
\end{aligned}
\tag{90}
$$

The arrows in this display indicate proof/DAG ordering. They are not unregistered converses. In particular, no scalar Ξ readout is used to reconstruct the retained source owner.

I.1 Independent review obligations

Table 6 gives the minimum independent attack surface. Each row can be disputed without granting the others. Operational review status is tracked outside the proof DAG in the OPEN `EXT-RH-*` Bugs Everywhere issues.

Table 6: External-review obligations for H-WIT002.

ID	Owner / object	Reviewer must establish
EXT-RH-001	H-WIT002 arbitrary zero	The input t is arbitrary over all complex zeros of the classical entire Ξ carrier, not sampled or preselected.
EXT-RH-002	H-VER156 cut-out	Its hypotheses apply to $\rho(t) = 1/2 + it$ on the already-owned actual-zero rail, without scalar-to-source reconstruction.
EXT-RH-003	common-r forcing	The H-AUD049/H-AUD076 forcing is universal over the retained actual-zero owner and all required criteria are present.

ID	Owner / object	Reviewer must establish
EXT-RH-004	H-VER164	The Ξ_{HG} carrier is genuinely global, bilateral, zero-safe and entire, rather than a sampled or half-plane identity.
EXT-RH-005	classical Ξ recognition	The Riemann–Pólya normalization and scale give the same entire carrier, including the same zeros and multiplicities.
EXT-RH-006	H-VER165	The O10 post-singleton join is complete on the signed family and never crosses the singleton barrier backwards.
EXT-RH-007	final affine algebra	For $t = a + ib$, $\mathrm{Re}(1/2 + it) = 1/2 - b$, so the H-VER156 endpoint forces $b = 0$ with no extra analytic premise.
EXT-RH-008	classical endpoint	The standard entire-Ξ real-zero statement is exactly equivalent to RH under $s = 1/2 + it$ [36, 40, 44].
EXT-RH-009	certified ancestry	Neither ordinary zeta nor Euler Gamma is load-bearing in the terminal H-WIT002 Section-0 ancestry.
EXT-RH-010	end-to-end reproduction	An independent reviewer can reconstruct the chain without trusting internal `CLOSED` labels, or can identify a precise named failing edge.

I.2 Hostile-review checklist

For each edge, the reviewer should record the quantifier and domain, whether the map is exact or lossy, whether a denominator or singular cell is introduced, whether multiplicity or sign data are discarded, and whether a downstream scalar is being used in reverse as a source premise. The reviewer should also compare the prose direction with the strict Section-0 DAG and with the current source manifest.

The detailed operational matrix, acceptance criteria, BE references and review entry points are maintained in

`docs/reviews/h-wit002-external-review-proof-map.md`

The OPEN review issues are deliberately not parents of H-WIT002. A review finding becomes load-bearing only if it is separately promoted under the normal theorem, DAG and BE protocols.

H-PAP010 publication target. The separate H-PAP010 manuscript is reserved for the terminal Xi-entire RH target which consumes the bilateral-entire H-VER164/H-PAP005 carrier. Status on 2026-10-29: H-WIT002 is closed again, restored on 2026-10-28 to its certification of 2026-10-04; the FULL40 column, H-WIT003, remains separately under audit, and the H-PAP010 manuscript remains a preregistration. The status Q&A of 2026-10-13 is:

`docs/qa/2026-10-13-h-pap010-purpose-and-status-qa.md`

The status-drift tracker is bug `#72f/2bff`.

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