

Abstract

This paper continues prior work [1] based on the insight that Rishon ultracoloured triplets (electron, up, neutrino in left and right forms) might simply be elliptically-polarised "mobius light". The important first step is therefore to identify the twelve (24 including both left and right handed forms) phases, the correct topology, and then to perform transformations (mirroring, rotation, time-reversal) to double-check which "particles" are identical to each other and which are anti-particle opposites. Ultimately, a brute-force systematic analysis will allow a formal mathematical group to be dropped seamlessly on top of the twelve (24) particles.

Contents

1	Introduction	1
2	Analysis	1
2.1	Chiral anti-particle identity: X	2
2.2	Z-axis mirror and time-reversal	3
2.3	Y-Axis operations	3
2.4	Neutrino weirdness	3
2.5	Jones vector superposition	4
3	Discussion	5

1 Introduction

The insight of the mobius optics analysis [1] is summarised in figure 1. It is however incomplete: there are both left and right-handed particles: two at each position. This does not affect the calculation of phase-transforms but it does not provide us with an explanation of why chirality exists, either.

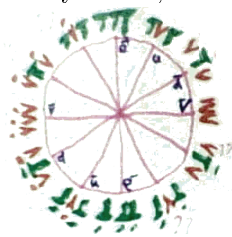


Figure 1: Twelve Rishon positions

To explain chirality we need something that works in three dimensions, and contains the possibility of preserving "spin half" characteristics in some way - hence the reason for exploring mobius strips. Figure 2 was generated counter-clockwise about the C point (Z-axis), whilst Figure 3 clockwise, but the elliptical-polarisation axis (represented by two points that start as a unit vector along the Z-axis) was rotated in the same way in both cases.

Other topologies were explored and eliminated, so we also need to show why the chosen topology works, that $e-R$ is also an $e+L$. that the up and down quark elliptical axes remain orthogonal (to create I-Frames), and also demonstrate that superposition of two particles maps to another particle, to the correct

$VT0$ phase transform in the case of gluons (aka pions) and in the case of leptons and baryons, to the correct "charge" (aka phase). Critical to this last analysis will be to ensure that chirality is correct (which simply cannot be done in 2D alone) or at least a rational explanation found.

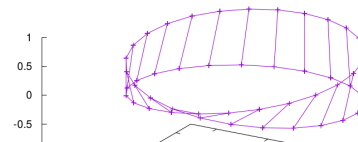


Figure 2: Left Rishon0 (e_L)

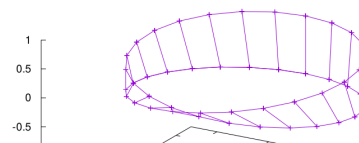


Figure 3: Right Rishon0 (e_R)

2 Analysis

Let $\tau = 2\pi!$ [16] As I lack the time and ability to comprehend the level of mathematics to carry out formal analysis, I decided to substitute trial-and-error and sheer brute force computation, developing python2 source [3] that would create the required list of points, display them using gnuplot so that they could be visually verified, generate the permutations needed and carry out the necessary transformations and comparisons which an experienced mathematician would easily do with formal proofs.

The first step was therefore to work out what set of mobius strips would represent the twelve (24) particles. I decided to skip the dual Gaussian Beam generation technique developed by Freund [5] and to go straight to plotting the rotation of the elliptical axes. A verbal representation of the generating algorithm is as follows, with cartesian coordinates (x,y,z):

1. start with ellipse rotation axis as Y axis (0,1,0)
2. rotate ellipse axis about Z by angle ω
3. Rishon-triplet starts upright (0,0,1)
4. Rishon-triplet is rotated about ellipse axis by 1/2 angle of C-point rotation, $\theta/2$.
5. At same time, Rishon-triplet is rotated about C point by angle θ

Two angles are involved here: ω is the angle associated with the particle type (as shown in figure 1), where θ is the angle used to generate 24 equally-spaced points around a circle from 0 to τ , generating for example the plots seen above. For a left chiral particle, that rotation is stepped negatively; right, positively.

One mistake that was made in the software actually discovered two possible potential configurations: steps 4 and 5 were inverted such that the ellipse axis was rotated relative to the line between the points and the C-Point (angle θ). Interestingly this did not make any difference to the results of the operations analysis, but one of the topologies may turn out to be incorrect when it comes to performing the Jones Vector superposition (later in this paper).

Various other topologies were tried, such as starting with the ellipse axis beginning at a vector along the X-Axis: it should be clear that this is merely a rotation of figure 1 through $\tau/4$. Starting as a vector along the Z-Axis resulted in a 2-dimensional object, definitely not a mobius, so were eliminated from enquiries. The ones that were *particularly* interesting were if the ellipse axis was up at an angle, for example, starting along the Y-Axis then rotated upwards by 45 degrees about X:

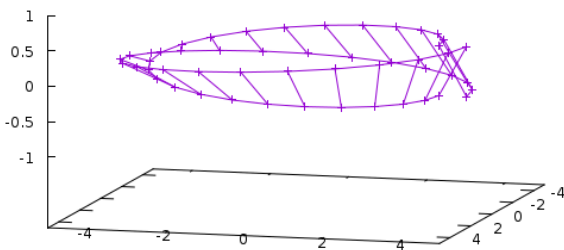


Figure 4: Starting axis 45 degree upright

Only one revolution has been carried out, here: the second revolution results in a second set of points where the elliptical axis is always orthogonal to where it was on the first revolution. This topology has not

been ruled out from enquiries, particularly given that when it comes to generating the neutrino and anti-neutrino, the 45 degree rotation *disappears* (in effect), leaving us with no "handed-ness" on the neutrino, but distinct "handed-ness" identity on all other particles. For now, however, I decided to go with the simpler topology (without the 45 degree addition) as the comparator-function of the python program is telling me that the simpler topology particles are still unique. The simpler topology also has the advantage that the angle does not change by 90 degrees on every half cycle of the mobius: given that we ultimately are talking about Maxwell's Equations here, that would represent a continuous oscillation of the E.M. field that feels... wrong somehow.

It was recently very kindly (and anonymously) explained to me the difference between a photon and an elliptical polarisation axis: the spin of a photon is represented by a vector, which returns only after a full rotation of 360 degrees, thus it may never generate a mobius strip. However, a line returns to itself after a 180 degree twist. This is slowly beginning to make sense to me, after accidentally creating a mobius that in effect bisects itself at right-angles all the way along its middle! So we leave this one for now.

It's also quite important to explain the operators of the program. An "applier" function was created that takes two sets of points, a list of operations (Rotate about X by 180 for example) and recursively brute-force searches for operations that may be applied to make the points of "set one" equal to "set two". Any branch of the recursive search that results in identity (e.g. two Rotations about X) anywhere along the set of operations applied so far, are pruned. Importantly: the comparator function performs its own rotational search about the Z-axis. This drastically reduces the recursive search space of the "applier" function.

2.1 Chiral anti-particle identity: X

One key task to perform is to verify that all particles are identical to their opposite-handed anti-particles. This was achieved by first writing a function named "search_equivalence" that takes two parameters: index of the phase ω , and chirality. It creates all possible particles (including anti-), and performs all possible transforms looking for the chosen target. Calling this function with an outer loop walking through all twelve particles resulted in output that could be visually confirmed. In each and every case, two operations were carried out: mirror about the X-axis and rotation about X by 180. The operator-applier function discovered in each case that these two operations could be applied in reverse-order.

A number of things need to be said about this result. Firstly: only three possible transforms were in-

cluded: Mirror about X, Rotate X 180, Time-Reverse. The reason for that is that it makes sense to limit the number of operations, otherwise the number of permutations rises enormously and unnecessarily. Including Mirror about Y would result in two mirror operations being applied, which is equivalent to another type of operation: the number of paths goes up to the exact same result. It is much simpler therefore to just limit the number of operations.

Secondly: the recursive depth was also kept down to three operations. This is to stop Time-reverse being applied at least twice. When the recursive depth was increased to six, a huge number of permutations occurred in which Time-reversal occurred exactly twice. This tells us that investigation of time-reversal should be kept out of the permutations, and done separately.

However: the Time-reversal operation was never applied when the depth was kept to a maximum of three. An additional test was therefore performed, to check if all chiral-reversed particles were equal to their anti-particle: turns out that they are. So *another* test was carried out, to see if MirrorX and Rotate X 180 were inverse operations for all particles (including chiral-reversed): turns out that that's the case, too.

The applier function did not spot this identity because it always performed one transform before doing any identity checks. Thus if the particles were *already* identical it would not notice. This is a result in itself because it tells us that the assumption of transforms exclusive to X are not necessarily going to have the same result as transforms in Y or Z.

2.2 Z-axis mirror and time-reversal

The arbitrary decision to restrict operations to X had the unintended side-effect of providing useful data but was shown to be slightly naive. Different operations were therefore chosen. The three permitted transformations are: Time-reversal, MirrorX (demonstrated above to be equivalent to be equivalent to a 180 Rotation about X), and Mirror about Z. Analysis of the results allows us to notice the following patterns:

- The three transforms are always all applied, in all six permutations, achieving the same net effect in all cases. Thus, we might (wrongly) conclude: Time-reversal is *equivalent* to the dual operations of mirroring in both Z and X simultaneously.
- $e-R$ is mapped to both $e-L$ and $e+R$ which we would expect because $e-L$ we have shown to be equivalent to $e+R$. We therefore filter out the anti-particle from further reporting
- The three transforms, when applied all together, appear to invert the Rishon clock (phase angles) whilst inverting the chirality. Thus, $1\tau/12_L$ is

transformed to $11\tau/12_R$, 2_L to 10_R and so on. In essence: Vohu is inverted, chirality is inverted, Tohu remains *the same*.

- The only location where this does *not* happen is for the neutrino and anti-neutrino, which is a fascinating result in its own right that warrants investigation and confirmation: $3\tau/12_R$ maps *only* to $9\tau/12_L$ (and likewise $9\tau/12_R$ only $3\tau/12_L$) whereas the electron $0\tau/12_L$ definitely maps to both $0\tau/12_R$ and $6\tau/12_L$.

Thus we may *not* naively assume that Time-reversal is equivalent to mirroring in both Z and X. Time-reversal appears to be a very weird operation. Up until I noticed that the position representing the neutrino is missing its anti-particle (which was a result we were kind-of expecting given that neutrinos are not supposed to have an inverted-chiral anti-particle), I would have said that Time-reversal is equivalent to the simultaneous combination of Z and X mirroring *and* Phase-mirroring (Vohu-sign-inversion) *and* chirality-inversion.

What is spectacularly weird is that we *just confirmed above* that the neutrino definitely inverts chirality when it inverts sign. A software bug is therefore suspected, which will be investigated next by close analysis of the actual points. If it turns out not to be a bug, then there is something phenomenally weird about Time-reversal.

2.3 Y-Axis operations

Thinking about the neutrino overnight I worked out what the problem was, and will describe it in a separate section. However before doing that it's worth exploring the Y-Axis operations. The first experiment was to chain RotateX-180, MirrorZ and MirrorY together. Fascinatingly, this was a *null result*. There were no combinations of these operations and particles that were equivalent.

The next attempt was to include MirrorZ, MirrorY and RotateY-180. This got results but none of them included MirrorZ. All operations where MirrorY and RotateY-180 were performed were permutations to the same particle. Just as with the X-Axis transforms they appear to map to the anti-particles. So, for example: whilst electron (L) $0\tau/12_L$ maps to the positron (R) $6\tau/12_R$, and the neutrino (L) $3\tau/12_L$ to the anti-neutrino (R) $9\tau/12_R$, and so on. Thus, again: Mirror-X plus Rotate-Y-180 are an anti-particle inverter operation.

2.4 Neutrino weirdness

The bug in the software was one that has already been encountered: comparison of particles that are already identical do not get picked up. That means that when

we compared $9\tau/12_R$ to $3\tau/12_L$ they were already identical, so the program did not tell us that. However with the function `check_neutrino_y_identity` we can conclude that even for the neutrino, the application of the three transformations (in any order) of Time-reversal, MirrorZ and MirrorX is a Vohu-inverting chirality-inverting function, where, due to the unique properties of the neutrino (zero Tohu) it happens also to be an *identity* operation as well.

This is a highly significant result as it means that the spectacular weirdness of the neutrino, from observations of particle decay patterns over the past few decades, is matched precisely by this strange mobius-light group and its operators.

2.5 Jones vector superposition

The next step is to look closely at the superposition of particles. This is extremely relevant for phase transformations (aka "decay") as well as showing that the compound particles (I-Frame [4]) superimpose to a stable pattern (one of the 12 phases), respect chiralty, and also that their constituents have orthogonal E.M. fields (right-hand motor rule is obeyed, basically).

The first idea that occurred was to go backwards through the maths to the original Gaussian beams of equation (1) in Freund's work [5]. The idea being: you take two pairs of generating beams, simulate them as being from exactly the same place, then it would be logical to simply team up the two GL0's together, separately from the two GL1's and do some simple sums. Turns out that working out the original generation beams requires Mathematica (or experimental equipment) so I dropped this idea and tried google searches "superposition elliptical polarized light" instead.

That gave me a clue that there is something called "Jones vectors", which may be expressed as:

$$E_{\hat{x}} = E_{0\hat{x}} e^{-i(kz - \omega t + \psi_{\hat{x}})} \quad (1)$$

So the initial question [15] was: how do you add Jones vectors, but then I recalled that there are actually two ways to generate the Rishon-triplet mobius paths. The first is to start with the (upright, Z-axis) unit vector on the Y-axis (0,-1, 0) and to rotate the elliptical polarisation rotation axis through the angle required to generate the particle. The second is to leave the elliptical polarisation rotation axis entirely alone and to *change (rotate) the starting point instead*.

The end result is exactly the same: in the latter case you simply rotate the entire particle anti-clockwise by the generator-angle, but the significant thing about the latter case is: when you compare two particles that are so aligned *the exponents from equation 1 are identical*. That in turn means that to superimpose two particles you simply **add up their vectors** $E_{\hat{x}}$.

However, that covers the case where the two particles are the same type, as in for example two "up quarks" of the proton. The third particle, the "down" quark, from prior investigations, has to have its E.M. polarisation axis at right-angles [1]. We therefore need to express the particles in terms of Jones vectors. Let the angle θ represent the particle phase, in n increments of $\tau/12$ where, from figure 1, the up quark is $n = 1$ and the down quark at $n = 8$. From the development of the python2 program we know that θ has to be added to the phase in the exponential part of the Jones Vector, that the elliptical rotation is at half speed, and that $E_{\hat{x}}$ is also dependent on θ . Bringing that all together and factoring the common parts (for example $kz - \omega t$) to outside we get an equation as follows:

$$E_{\hat{x}} = E_{n\hat{x}} e^{-i(kz/2)} e^{-i(-\omega t/2)} e^{-i(\theta/2)} \quad (2)$$

where $-\omega t/2$ is the rotation over time of the elliptical polarization axis, and where the relationship between each of the particle's elliptical polarisation axes may be expressed as:

$$E_{n\hat{x}} = E_{0\hat{x}} e^{-i(\theta)}, \theta = n\tau/12 \quad (3)$$

For two particles to phase-synchronise, i.e. all these exponents to "line up", and thus present us with the circumstances where the axis vectors may be added, the phase angle of the particles has to either be equal, or opposing, or *double*. (*Note: this likely explains why there are only 12 possible phases, as 12 divisions is the only way in which these strange criteria may be satisfied*).

The up quark is at $n = 1$ and the down quark at $n = 8$, which previously gave us a bit of a problem. In the prior paper in which this conundrum occurred [1] we surmised that the down quark needed to be rotated about the Z Axis and the X Axis by 180 degrees. If however we have two equations of type 3 above where one is half the angle of the other, it should be plainly obvious that offsetting (rotating) each particle by appropriate angles that are twice the other - for example $\tau/2$ and $\tau/4$ - will result in the exponent parts having a common factor which allows the Jones Vectors to be superimposed (summed), and the remaining factors which are not common will be rotation operations of 180 degrees and thus also cancel out.

Therefore we have circumstances where the pion's up and anti-down quarks (being phases $1\tau/12$ and $2\tau/12$ respectively) can be rotated to be $3\tau/12$ apart yet the Jones Vector exponential parts have a common factor allowing superposition; likewise with up and down quarks of the proton, and also just as importantly, likewise with the electron-positron composition of the muon (although this case is more obvious than that of the up and down quarks).

I cannot emphasise enough how incredibly significant this insight is, because it means that we have a genuine reason as to why the Rishon Model's "Tohu" and "Vohu" have always been derived as simple summation. It's not a mistake as I previously surmised: it's the **correct** thing to do, because Tohu and Vohu are inherently encoded into the $E_{n\hat{x}}$ axis of elliptical polarization in phase-coherent superimposed non-paraxial mobius-light.

And if the summation of the vectors is the correct thing to do, then all the analysis of the Extended Rishon Model has a sound theoretical basis, *including* the issue of the pion+ being unstable, because the superposition of the two quarks result in both electrical field presence *and* magnetic field presence (Tohu and Vohu both summing to 1 or -1 simultaneously), *not* as we previously surmised [1] the *generating* angles needing to be added.

Chirality, then, is relevant because only rotationally-matched particles may have their $E_{\hat{x}}$ vectors superimposed. Thus we also have a justification for $VT0+\overline{VT}0$ pairs being both different from but crucially incompatible with $V\overline{T}0+\overline{V}T0$ phase-transform operations [1]. This should be expanded, clarified and explored separately, as it is a large topic on its own.

Also: as noted the mistake made in a prior revision of the software, where the elliptical axis was accidentally rotated about the C Point: whilst topologically it's fine, it prevents and prohibits the Jones Vectors from lining up (there's an extra term in the exponential part of the Jones Vector). We therefore can eliminate this mistaken topology from enquiries.

3 Discussion

It's actually very challenging for me not to get too hugely excited by the insights noted in this paper: not least that there still isn't anything that contradicts the Extended Rishon Model but instead confirms it, but that the various misconceptions (including those that I made) about the Rishon Model are clarified and explained.

- Sundance O Bilson-Thompson [9] was correct in his insight to explore the Rishon Model as a topological model, including mobius twists to represent preons, but without the insight of the mobius elliptical-polarisation axes it doesn't provide a direct link to Maxwell's equations.
- Piotr Zenczykowski [8] moves Rishons to O(6) phase space and uses Clifford Algebra to work on them, but unfortunately again without the key of the mobius-light, the group he creates has no clear direct link that I can discern to mobius-light, instead his work successfully relies on a tie-in to SU(3)xU(1) (which we noted earlier does

not recognise the concept of Vohu). It is however very important to note that the group he developed may actually turn out to be right one as it is also at heart a *geometric* group.

- Harari [6], myself, Zenczykowski, Bilson-Thompson and *everyone* who has ever worked on the Rishon Model created rules involving the vector summation of Tohu and Vohu, none of us realising that that's the correct thing to do *only* if it's riding in effect on the back of a phase-sync'd "Mobius rollercoaster".

The mappings discovered (so far) are also worth listing in summary form to make it easier to identify a potential group:

- Inversion of all characteristics (Inverse Tohu, Vohu and Chirality) is the identity.
- Rotation-180-X plus Mirror-X result in identity, Rotation-180-Y plus Mirror-Y likewise.
- There is no mapping on Rotation-180-X and Mirror-Y to any particles
- There is no mapping on Rotation-180-Y and Mirror-X to any particles
- Time-reversal and the combined operations of "Mirror-Y and Mirror-Z" create an inverse-chiral particle with a mirrored phase-generating angle (see figure 1). In effect: chirality and the sign of "Vohu" are both inverted but Tohu is **not**.

This latter leads to the extremely relevant insight that it's a unique property of the mobius topology that weirds-out the phase transforms exclusively for the neutrino, leaving an anti-symmetrical aspect in particle "decay" that now has a potential explanation.

Regarding phase transforms (aka "decay") we now have an explanation as to why the (provisionally-named) over and under quarks [1] from which one type of gluon (aka ultra-short-lived pion) are made are not involved in the standard left-handed universe "decay" patterns: the opposing direction of the exponent part of a Jones vector (equation 1) means that the particles have no way in which to phase-sync in order to effect the vector summation and complete the phase-transfer to the new "decay" result(s). It *really is* important to preserve chirality but there's simply been no other prior model that has such a simple (if weird) foundation based (ultimately) in Maxwell's Equations.

Pions of both positive and negative charge in particular (all the way up to the W and Z Bosons [4]) we have confirmed that they do indeed have Tohu and Vohu at +/-1 simultaneously, where an example of the first half of such a path configuration is shown in figure 4 (total magnitude being $\sqrt{2}$ due to Tohu and Vohu $E_{\hat{x}}$ vector contributions from each quark both being

of magnitude 1). Also the fact that the two quarks up and anti-down are 90 degree phase apart (when one of them is rotated upside-down) means that their paths *can* be orthogonal, the exponent part of their Jones vector *can* be precisely phase-sync'd, leading to the elegant and beautiful satisfying of all of the seemingly-contradictory preconditions. Leptons likewise, in both I-Frame and Dumbell configurations [4].

It is also not contradictory to treat the separate particles as "braided light" [10], in fact it is a fundamental requirement to do so. In the case of the proton, the two up quarks are in fact *held together* in close proximity by the down quark being orthogonal to both of them, with the two down quarks, we surmise, being *approximately* (near fully) superimposed. It is noted with significance Freund's comment [5] that in simulations that mobius-light remained stable even when 3 percent noise was added.

Also it is with some huge relief that the Jones vector insight was found. Without that, it would have undermined (called into question) the prior work done into phase transforms [7] [4], requiring that it be re-evaluated.

There is still an enormous amount of work left to be done, here. An incomplete list would be:

- Finding a formal mathematical group that maps onto the operators and 24 particles. Candidates include Lorentz Groups, $SO(3) \times SU(2)$, Lie Groups, $SU(3,1)$, $O(6)$ phase-space with Clifford Algebra [8], and many more.
- Extending the python2 program to add a "superposition" binary operator, with the intent to use that to confirm which particles, when added, map to others: confirmation of the VT^* phase-transform operations, basically.
- Are over and under really undiscovered quarks? Attempting to perform matched $VT0-\bar{VT}0$ transforms on both left-handed and right-handed particles will help confirm if they are genuinely new quarks or if they are synonymous with oppositely-chiral up and down quarks. We suspect the former but this needs to be tested by trying out some phase-transforms (aka "decay").
- Working "mass" into the equation in some way, now that there is a candidate actual geometric underpinning based in Maxwell's Equations for *inside* particles. I suspect that given that the mobius strips are circular, it may be as simple as tying in to Dr Randell Mill's work ("Great Circles", "Orbitspheres" [11]) or that of Andrew Worsley [12] or Jay Yablon [13].
- Working out how small mobius strips (small radius) could transform into larger strips (larger radius). Is it as simple as how an electron skips

to a higher orbit? Or, does the EM wave of each quark remain oscillating at a fundamental frequency (Compton wavelength) but when in a Jones-superposition arrangement there are many more "twists", thus preserving the Compton wavelength of oscillation and simultaneously preserving for example the proton's radius?

- Could the mobius insights discovered here be so soundly applied to other models of particles? Is it just a coincidence that mobius elliptically-polarized light happens to have the properties that satisfy the demanding preconditions? We suspect not but cannot rule out the possibility, particularly in light of Kaminer et al's work [14]

I think it's really important to mention why I believe the insight about Jones vector superposition has not been noticed before (used to such elegant effect in any particle model), and it's simply down to the fact that when the exponent part(s) align, it's a simple summation of vectors, but the key is: this *only* works in the unique case of mobius topology. It is therefore *really easy* to miss that, and instead empirically derive something like the original Rishon Model or (in the case of theories where Vohu, representing the "magnetic" component, does not feature at all) the Standard Model, as being based purely on vector summation "rules" and "conservation", instead of a more elegant hybrid stemming from the unique properties of mobius strips.

In addition to that, the Standard Model, by moving all investigations into the frequency domain (by way of Yang-Mills being a generalisation of Maxwell's Equations moved to Quantum Mechanics, which is known to be a form of Fourier Analysis) has unfortunately completely missed the opportunities for investigation of particles potentially being purely phase-coherent arrays of photons. Relationships such as inter-dependent (fixed) phase angles, when moved to the frequency domain, are constants that are *dropped*, similar to where constants drop out of differentiation. *Such critical information cannot be recovered.* Even more unfortunately than that: because the way that Quantum Mechanics has been used to view particles exclusively from the frequency domain, things like Heisenberg's Uncertainty Principle **actively prevent and prohibit** theoreticians from investigating or considering what might be inside particles: collaboration which could lead to a breakthrough is almost out of the question as the collaborators would need to fight their way out of the perspective traps created ironically by the successful use of QM!

Returning to Maxwell's Equations, and applying the knowledge learned over the past few decades in the field of optics, seems to be getting results. Sadly, it is a different fundamental basis to the Standard Model.

References

- [1] Luke Kenneth Casson Leighton, *A further exploration of the preliminary implications of Hypercolour as being phase-order of "mobius" elliptically-polarised light in the Extended Rishon Model* vixra:1701.0316, 03 Jan 2017.
<http://vixra.org/abs/1701.0316>
- [2] Luke Kenneth Casson Leighton, *Exploring the Addition of Hypercolour to the Extended Rishon Model*, vixra:1701.0290, 03 Jan 2017.
<http://vixra.org/abs/1701.0290>
- [3] Python2 source code (GPLv3+ licensed) for the Rishon Group Analysis
http://lkcl.net/reports/rishon_group_analysis/
- [4] Luke Kenneth Casson Leighton, *A Discourse on the Electron and Other Particle's Internals, from the Perspective of the Extended Rishon Model and the Field of Optics*, vixra:1701.0016, 03 Jan 2017.
<http://vixra.org/abs/1701.0016>
- [5] Isaac Freund, *Multi-twist optical Mobius strips*, arxiv:0910.1663, 9 October 2009.
<http://arxiv.org/abs/0910.1663>
- [6] Harari, H. (1979). *A Schematic Model of Quarks and Leptons*, Physics Letters B 86 (1): 83–86. Bib-code:1979PhLB...86...83H. doi:10.1016/0370-2693(79)90626-9.
- [7] Luke Kenneth Casson Leighton, *An introduction to the Extended Rishon Model*, vixra:1403.0016, 03 March 2014,
<http://vixra.org/abs/1403.0016>.
- [8] Piotr Zenczykowski (2008), *The Harari-Shupe preon model and nonrelativistic quantum phase space*, arXiv:hep-th/0803.0223
- [9] Sundance Osland Bilson-Thompson, *"A topological model of composite preons"*, arXiv:hep-ph/0503213, 2005.
- [10] Thomas Iadecola, Thomas Schuster, and Claudio Chamon, *Non-Abelian Braiding of Light*, DOI:10.1103/PhysRevLett.117.073901, arXiv:1509.05408, 10 August 2016
<http://phys.org/news/2016-08-physicists-method-braiding.html>,
- [11] Dr Randell L Mills, *The Grand Unified Theory of Classical Physics*, September 2016 Edition.
<http://brilliantlightpower.com/book-download-and-streaming/>
- [12] Dr Andrew Worsley, *A Chronicle of Modern Physics, Book 2, Flying on the Wings of Genius*. 2006.
- [13] Jay R Yablon, *Why Baryons Are Yang-Mills Magnetic Monopoles*, vixra:1211.0151, January 2013 in Number 4, Volume 35, 2012 pp 401-468 of the Hadronic Journal.
<http://vixra.org/abs/1211.0151>
- [14] Rivka Bekenstein, Jonathan Nemirovsky, Ido Kaminer and Mordechai Segev, *Nondiffracting Accelerating Wave Packets of Maxwell's Equations*, doi:10.1103/PhysRevLett.108.163901, 6 April 2012.
- [15] Stackexchange question about superposition of light. With much gratitude to Rod Vance for the kind assistance and valuable answer.
<http://physics.stackexchange.com/questions/305010/>
- [16] <https://tauday.com/tau-manifesto>